Resonance Width Consideration for Compton Rotation in Magnetic Field

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Abstract

In the framework of tree approximation a formula is obtained for the calculation of Compton rotation angle of the plane of linear polarization of photons per unit path in electron gas with high degree of spin polarization of electrons. The finite width of resonance on intermediate virtual Landau level is taken into account, averaging on the momenta of electrons is performed in zero temperature approximation.

1 Introduction

In present work the research is done according to the suggestion of V.G. Baryshevsky and V.V. Tikhomirov. The effect of Compton rotation of the plane of polarization of hard X-(soft gamma-) photons in the absence of magnetic field was theoretically predicted by V.G. Baryshevsky and V.L. Luboshitz in 1965 and experimentally tested at early 1970s [1, 2, 3, 4].

In [5], using the approach of [6], a formula was obtained for the calculation of Compton rotation angle of the plane of linear polarization of photons per unit path in electron gas with total spin polarization of electrons ($p_{0e} = 1$), though the finite width of resonance on intermediate virtual Landau level was not considered:

$$\frac{d\varphi}{dl} = \frac{(\pi\hbar c)^2 n_e \alpha \cos\beta}{\hbar\omega(\varepsilon_0 + \hbar\omega)} \exp\left(-\frac{\phi}{2}\right) \sum_{n=1}^{\infty} \phi^{n-1}(\Xi_n(g) - \Xi_n(f)), \alpha = \frac{e^2}{\hbar c},$$

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$$\phi = \frac{\hbar\omega^2 \sin^2\beta}{cBe}, \Xi_n(\lambda) = \frac{c\lambda_0\varepsilon_0 - \lambda_3 p_z c^2 - m^2c^4}{c^2\lambda_0^2 - \varepsilon_{n\lambda}^2 - i\cdot 0},$$
$$\varepsilon_0 = \sqrt{m^2c^4 + p_z^2c^2}, \varepsilon_{n\lambda} = \sqrt{m^2c^4 + 2ne\hbar Bc + \lambda_3^2 c^2}. \quad (1)$$

Here n_e is electron density, m, p_z are electron's mass and momentum along z axis, respectively; μ_B is Bohr magneton, e is electric charge, $\hbar\omega$ is photon's energy, $\vec{B}$ is magnetic field strength, β is the angle between the wave vector of photon $\vec{k}$ and $\vec{B}$; $\varepsilon_{n\lambda}$ is energy of virtual electron on intermediate nth Landau level. For such an electron in R-process ($\lambda = g$) and S-process ($\lambda = f$) we have

$$cg_0 = \varepsilon_0 + \hbar\omega, cg_3 = p_z c + \hbar\omega\cos\beta, cf_0 = \varepsilon_0 - \hbar\omega, cf_3 = p_z c - \hbar\omega\cos\beta. \quad (2)$$

Formulas (1) are applicable if spin polarization of electrons $p_{0e} = 1$ which corresponds to

$$2^{2/3} m\mu_B B \geq \pi^{4/3}\hbar^2 n_e^{2/3}. \quad (3)$$

2 Consideration of resonance width on Landau level

If $\hbar\omega < \varepsilon_0$ (which corresponds to X-photons), then the width Γ_n of resonance on nth Landau level in R-process must be considered, but the resonance in S-process is absent. Removing the poles from the real axis in $\Xi_n(g)$ and extracting the real part, we obtain

$$\varepsilon_{ng} \to \varepsilon_{ng} - i\Gamma_n/2, \Gamma_n \approx \frac{16(2n-1)\alpha(\mu_B B)^2}{3mc^2}, \quad (4)$$
$$Re(\Xi_n(g)) = \frac{(cg_0\varepsilon_0 - g_3 p_z c^2 - m^2c^4)G_n}{G_n^2 + \Gamma_n^2\varepsilon_{ng}^2}, G_n = c^2 g_0^2 - \varepsilon_{ng}^2 + \frac{\Gamma_n^2}{4}. \quad (5)$$

Numerical calculations show that similar consideration of Γ_n in S-process is not important, at least, at $\hbar\omega < mc^2$.

3 Averaging of amplitudes over momenta at T=0 K

Let's average (1) over electron momenta p_z at T=0 K, using the formula for electron state density per unit volume [7] and the formula for the averaging of an arbitrary function f:

$$\frac{dN_e}{V} = \frac{eBdp_z}{(2\pi\hbar)^2 c} \Rightarrow \langle f \rangle = \frac{1}{2w_1}\int\limits_{-w_1}^{w_1} f(w)dw, w = \frac{p_z}{mc}, w_1 = \frac{\pi^2(\hbar c)^3 n_e}{(mc^2)^2\mu_B B}, \quad (6)$$

$$\frac{d\varphi}{dl} = \frac{e^2 m\mu_B B cos\beta}{4\hbar^3\omega} \exp\left(-\frac{\phi}{2}\right)\sum_{n=1}^{\infty}\phi^{n-1}\int\limits_{-w_1}^{w_1}(R_n - S_n)dw,$$

$$R_n = \frac{f_1(w)f_2(w)}{f_3(w)\left(f_1^2(w) + \frac{\Gamma_n^2}{\hbar^2\omega^2}\left(1 + 4n\frac{\mu_B B}{mc^2} + (w + tcos\beta)^2\right)\right)},$$

$$S_n = \frac{1}{f_3(w)}\left(1 - \frac{Q_n}{Q_n - f_2(w)}\right), Q_n = tsin^2\beta - 4n\frac{\mu_B B}{\hbar\omega},$$

$$f_1(w) = Q_n + f_2(w) + \frac{\Gamma_n^2}{4\hbar\omega mc^2}, f_2(w) = 2(\sqrt{1+w^2} - wcos\beta),$$

$$f_3(w) = \sqrt{1+w^2} + t, t = \frac{\hbar\omega}{mc^2}. \quad (7)$$

The integrals can be taken numerically or analytically, and for S-process we obtain (analytical expressions for R-process are very complicated):

$$S_n = 2arsh(w_1) - \frac{4ty_1}{\sqrt{1-t^2}} - Q_n I_n, \quad (8)$$

$$2I_n(1-t)^2 cos\beta = (1-t)y_3 - 2t\left(\sigma_2 y_3 cos\beta - \frac{2\sigma_2 y_1}{\sigma_1}\right), Q_n = -2; \quad (9)$$

$$I_n = \frac{4t}{\Omega_{1n}}\left(\gamma_{1n}\left(\frac{4\xi_n y_2}{\nu_n} + ln\left|\frac{\tau_{n+}}{\tau_{n-}}\right|\right) - \frac{8\gamma_{2n}y_2}{\nu_n} + \frac{2\gamma_{3n}y_1}{\sigma_1}\right) +$$

$$+\frac{16y_2}{(1-t)(Q_n+2)\nu_n}, Q_n \neq -2, \mu_n = 0; \quad (10)$$

$$I_n = \frac{1}{\Omega_{1n}}\left(\frac{8t\gamma_{3n}y_1}{\sigma_1} + 2t\gamma_{1n}ln\left|\frac{y_2^2 + \xi_n y_2 + q_n}{y_2^2 - \xi_n y_2 + q_n}\right|\right) +$$

$$+\frac{2\Omega_{2n}}{\Omega_{1n}}\left(arctg\left(\frac{\tau_{n+}}{\sqrt{|\mu_n|}}\right) - arctg\left(\frac{\tau_{n-}}{\sqrt{|\mu_n|}}\right)\right)\theta(\mu_n) +$$

$$+\frac{\Omega_{2n}}{\Omega_{1n}}ln\left|\frac{\left(2y_2 - \sqrt{|\mu_n|}\right)^2 - \xi_n^2}{\left(2y_2 + \sqrt{|\mu_n|}\right)^2 - \xi_n^2}\right|\theta(-\mu_n), Q_n \neq -2, \mu_n \neq 0; \quad (11)$$

$$\sigma_1 = \sqrt{\frac{1+t}{1-t}}, \sigma_2 = \frac{cos\beta}{\sigma_1^2 cos^2\beta + 1}, \xi_n = -\frac{4cos\beta}{2+Q_n}, q_n = \frac{2-Q_n}{2+Q_n},$$

$$\gamma_{1n} = \frac{\xi_n}{\gamma_{4n}}, \gamma_{2n} = \frac{\xi_n^2}{\gamma_{4n}} - \gamma_{3n}, \gamma_{3n} = \frac{q_n - \sigma_1^2}{\gamma_{4n}}, \gamma_{4n} = (q_n - \sigma_1^2)^2 + \xi_n^2\sigma_1^2,$$
$$\nu_n = 4y_2^2 - \xi_n^2, \mu_n = 4q_n - \xi_n^2, \tau_{n\pm} = \xi_n \pm 2y_2,$$
$$\Omega_{1n} = (1-t)^2(Q_n+2), \Omega_{2n} = \frac{2(2\gamma_{2n} - \xi_n\gamma_{1n} + 1)t - 2}{\sqrt{|\mu_n|}},$$
$$y_1 = arctg\left(\frac{y_2}{\sigma_1}\right), y_2 = \frac{w_1 + \sqrt{1+w_1^2} - 1}{w_1 + \sqrt{1+w_1^2} + 1}, y_3 = ln\left|\frac{1 - y_2 cos\beta}{1 + y_2 cos\beta}\right|. \quad (12)$$

Here $\theta(\eta)$ is Heaviside function.

4 Numerical results and applications

For example, if $n_e \sim 10^{23}$ cm^{-3}, $\hbar\omega \sim 0.1$ MeV (hard X-region), $B \sim 10^{13}$ Gs (close to the pole of a neutron star), then $d\varphi/dl \sim 10^2 - 10^3$ rad/cm at the resonance. Besides: 1. Compton rotation can change its sign (like nuclear spin precession of neutrons [1]), that's why the addition of Compton and Faraday rotation can give zero at some ω, B. 2. The increase of β leads to the decrease of the resonant frequency ω_R, but the increase of B leads to the increase of ω_R; in both cases the value of $d\varphi/dl$ decreases at the resonance. In order to obtain more realistic formula for $d\varphi/dl$, finite temperatures must be considered, though the procedure of averaging over p_z becomes more complicated.

Compton rotation is important in some astrophysical problems. One can estimate n_e and the degree of ionization of cosmic plasma from the formula for $d\varphi/dl$ where other quantities can be measured or estimated: 1. The value of ϕ can be measured at different ω. 2. B, T can be estimated by different methods. 3. If $B > 10^{10}$ Gs, then the degree of ionization is about unity, almost all electrons are free and Faraday rotation is absent; otherwise,if $B < 10^9$ Gs, the difference of estimations of n_e from the formulas for $d\varphi/dl$ in cases of Faraday and Compton rotation helps to estimate the degree of ionization.

Besides, photon magnetic splitting becomes very important at $B \sim 10^{12} - 10^{13}$ Gs, that's why Compton rotation dominates at $B \sim 10^{10} - 10^{11}$ Gs in comparison with photon splitting and Faraday rotation.

5 Summary. The main results

Considering the finite width of resonance on intermediate Landau level for a virtual electron, in the framework of tree approximation a formula is obtained for the calculation of Compton rotation angle of the plane of linear polarization of photons per unit path in electron gas with high degree of spin polarization of electrons. Averaging on the momenta of electrons is performed in zero temperature approximation, where integration is performed for S-diagram of Compton scattering.

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