Nonlinear Dynamics and Applications

Minsk 2022

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June 21-24, 2022, Minsk, Belarus
Fractals, Chaos, Phase Transitions, Self-organization

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V.A. Shaparau
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Preface

The 29th Anniversary International Seminar (June 21-24, 2022, Minsk, Belarus) traditionally had subsubjects: "Fractals, Chaos, Phase transitions, Self-organization", plenary session and the following section sessions: Particles, Modelling and Safety Related Analyses of NPP, Quantum and Classical Electrodynamics, Gravity, Mathematics and Fields, Medicine, Biological and Chemical systems. 20 plenary, 37 section and 16 poster reports were submitted to the 29th Seminar by the scientists. At this Anniversary Seminar, the participants gave overview reports, which became useful to young scientists. Thus, in addition to the scientific component, the seminar also played an educational role. Most of the papers were included into these Proceedings.

Local Organizing Committee (A.G. Trifonov (Chairman), V. A. Shaparau (Vice-Chairman), L. F. Babichev, V. G. Baryshevsky, I. D. Feranchuk, M. V. Galynski, T. N. Korbut, G. G. Krylov, Yu. A. Kurochkin, A. V. Pestsova (Secretary), V. M. Red'kov, V. A. Savva, E. A. Shaparava (Scientific Secretary), R. G. Shulyakovsky) thanks colleagues Professors Yu. L. Bolotin, I. M. Dremin, V. A. Gaisenok, R. C. Hwa, L. L. Jenkovszky, N. S. Kazak, S. Ya. Kilin, W. Kittel, A. V. Kuzmin, S. A. Maksimenko, P.V.E. McClintock, M. Robnik, N. F. Shul’ga, A. L. Tolstic, G. M. Zinoviev for their valuable advises.

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To the problem of the contribution of spin-polarized protons to Baryshevsky–Luboshits effect

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Baryshevsky-Luboshits effect is possible not for spin polarized electrons only. A similar possible effect is predicted for spin polarized protons. The general structure of the formula for the angle of rotation of the plane of polarization of a photon moving among spin polarized protons depends on the difference of Compton scattering cross sections for the same and opposite spin directions of a photon and a proton. The contribution is obtained to such a formula depending on the integral of the difference of the corresponding cross sections. The final formula contains terms with double, triple and quadruple integrals. The problem is of interest for astrophysics because the possibility of spin polarization of protons is predicted for some types of stars (white dwarfs, Supernovae of type II), and if leads to the problem of the contribution of protons to Baryshevsky–Luboshits effect in comparison with spin polarized electrons.

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Keywords: Baryshevsky–Luboshits effect, Compton forward scattering amplitudes, spin polarization of protons, white dwarfs, Supernovae II explosions.

1. Introduction

Baryshevsky-Luboshits effect, predicted in 1965 and experimentally discovered in 1970s [1], is the rotation of photon polarization plane in spin-polarized electron gas explained by the difference of Compton scattering amplitudes for the same and opposite spin directions of photon and electron.

Another effect, similar to Baryshevsky–Luboshits effect on spin polarized electrons, is predicted for matter with spin-polarized protons due to the possibility of Compton effect on protons.

The angle of the rotation of photon polarization plane per unit path depends on the photon frequency, the degree of spin polarization of particles (electrons or protons), the particle number density, the photon wave vector direction relative to the spin polarization vector and the real part of Compton forward scattering amplitude on the corresponding particle.

We consider the general structure of the expression for the real part of Compton forward scattering amplitude to be the same for an electron and a proton. It means that the expression contains the term depending only photon frequency and the anomalous magnetic moment of the corresponding fermion and the term depending on the difference

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of Compton scattering cross sections for the same and opposite spin directions of a photon and a fermion.

The expressions for both terms are well-known in case of electrons [1]; the expressions for the term depending only on photon frequency and the anomalous magnetic moment of a proton or a neutron were obtained in [2]. It is interesting to obtain the expression for the term depending on the difference of Compton scattering cross sections for the same and opposite spin directions of a photon and a proton. The problem can have astrophysical applications because spin polarization of nucleons is predicted in astrophysics also [3].

2. General formula for Baryshevsky–Luboshits effect on polarized protons

If a photon passes the distance dx in proton gas and its plane of linear polarization turns on the angle $d\varphi$, the following denotations will be used for $\frac{d\varphi}{dx}$ calculation:

n_p – proton number density, $\mathbf{p}_p$ — proton spin polarization vector, ω — photon frequency, $\mathbf{n}$ — unit vector in the direction of photon propagation, c — speed of light in vacuum.

The general formula for $\frac{d\varphi}{dx}$ calculation in proton gas is similar to the corresponding formula obtained for electron gas [1] and can be written in the following way [2]

$$\frac{d\varphi}{dx} = \frac{2\pi n_p c}{\omega} \left(\mathbf{p}_p \cdot \mathbf{n}\right) \mathrm{Re} f_p\left(\omega\right) , \tag{1}$$

where $f_p\left(\omega\right)$ is the spin-dependent part of Compton forward scattering amplitude on a proton.

3. The general expression for the real part of Compton scattering amplitude

Let's assume the general structure of $\mathrm{Re} f_p\left(\omega\right)$ to be the same as the structure of the spin-dependent part of Compton forward scattering amplitude on an electron. Then we can write [1]

$$\mathrm{Re} f_p\left(\omega\right) = s\left(\omega, \Delta\mu_p\right) + \frac{2\omega}{\pi} \int_0^{+\infty} \frac{\mathrm{Im} f_p\left(y\right) dy}{y^2 - \omega^2} . \tag{2}$$

The expression for $s\left(\omega, \Delta\mu_p\right)$, where $\Delta\mu_p$ is the anomalous magnetic moment of a proton measured in nuclear magnetons, was obtained in [2]. The expression for $\mathrm{Im} f_p\left(y\right)$ is

$$\mathrm{Im} f_p\left(\omega\right) = -\frac{y}{8\pi c} \left(\sigma_{\uparrow\uparrow}\left(y\right) - \sigma_{\uparrow\downarrow}\left(y\right)\right) , \tag{3}$$

where $\sigma_{\uparrow\uparrow}\left(y\right)$ and $\sigma_{\uparrow\downarrow}\left(y\right)$ are Compton cross sections for the same and opposite directions of spins (helicities) of a photon and a proton, respectively.

According to (2) and (3) we obtain

$$\mathrm{Re} f_p\left(\omega\right) = s\left(\omega, \Delta\mu_p\right) - \frac{\omega}{4\pi^2 c} \int_0^{+\infty} \frac{y\left(\sigma_{\uparrow\uparrow}\left(y\right) - \sigma_{\uparrow\downarrow}\left(y\right)\right) dy}{y^2 - \omega^2} . \tag{4}$$

The expression for $\sigma_{\uparrow\uparrow}(\omega) - \sigma_{\uparrow\downarrow}(\omega)$ is relatively simple for electrons that's why the integral in (4) can be calculated in elementary functions except one term which is calculated numerically [1]. The expression for $\sigma_{\uparrow\uparrow}(\omega) - \sigma_{\uparrow\downarrow}(\omega)$ is much more complicated for protons and can be presented in the form

$$\sigma_{\uparrow\uparrow}(\omega) - \sigma_{\uparrow\downarrow}(\omega) = \int \left(\frac{d\sigma_{\uparrow\uparrow}(\omega)}{d\Omega} - \frac{d\sigma_{\uparrow\downarrow}(\omega)}{d\Omega} \right) d\Omega \,, \tag{5}$$

where $d\Omega$ is solid angle

$$d\Omega = 2\pi \sin\theta d\theta \,. \tag{6}$$

4. The expression for the difference of the differential cross sections

The expression for $\frac{d\sigma_{\uparrow\uparrow}(y)}{d\Omega} - \frac{d\sigma_{\uparrow\downarrow}(y)}{d\Omega}$ is the following [4]:

$$\frac{d\sigma_{\uparrow\uparrow}(y)}{d\Omega} - \frac{d\sigma_{\uparrow\downarrow}(y)}{d\Omega} = -\hbar^2 c^2 \frac{2\alpha^2 m_{np} c^2}{m_{np}c^2 + 2E_\gamma} \sum_{i=1}^{4} f_i(y,\theta) \;, \tag{7}$$

$$f_1(y,\theta) = A_3^2(y,\theta)\sin^2\theta \,, \tag{8}$$

$$f_2(y,\theta) = A_1(y,\theta)\, A_3(y,\theta) \left(1 + \cos^2\theta\right) \;, \tag{9}$$

$$\begin{aligned} f_3(y,\theta) = \hbar^2 y^2 \sin^2\theta (A_6(y,\theta)\,(A_1(y,\theta) + 3A_3(y,\theta)) + \\ + \cos\theta (3A_3(y,\theta)\, A_5(y,\theta) - A_1(y,\theta)\, A_5(y,\theta) + \\ + A_3(y,\theta)\, A_4(y,\theta) - A_2(y,\theta)\, A_3(y,\theta)))\,, \end{aligned} \tag{10}$$

$$\begin{aligned} f_4(y,\theta) = \hbar^4 y^4 \sin^2\theta (A_5(y,\theta)\,(A_2(y,\theta) - A_4(y,\theta))\sin^2\theta + \\ + 4A_5(y,\theta)\, A_6(y,\theta)\cos\theta + 2A_6^2(y,\theta) + 2A_5^2(y,\theta)\cos^2\theta)\,, \end{aligned} \tag{11}$$

$$\begin{aligned} A_1(y,\theta) = -\frac{1}{m_{np}c^2} + B(m_\pi c^2 - \sqrt{m_\pi^2 c^4 - \hbar^2 y^2} + \\ + \left(\frac{1}{2}\arctan\frac{\sqrt{-t(y,\theta)}}{2m_\pi c^2} - \int_0^1 \arctan\frac{(1-z)\sqrt{-t(y,\theta)}}{2\sqrt{m_\pi^2 c^4 - \hbar^2 y^2 z^2}} dz \right) \frac{2m_\pi^2 c^4 - t(y,\theta)}{\sqrt{-t(y,\theta)}})\,, \end{aligned} \tag{12}$$

$$\begin{aligned} A_2(y,\theta) = \frac{1}{m_{np}c^2\hbar^2 y^2} + \\ + B\frac{t(y,\theta) - 2m_\pi^2 c^4}{(-t(y,\theta))^{3/2}} \int_0^1 (\arctan\frac{(1-z)\sqrt{-t(y,\theta)}}{2\sqrt{m_\pi^2 c^4 - \hbar^2 y^2 z^2}} - \\ - \frac{2(1-z)\sqrt{t(y,\theta)(\hbar^2 y^2 z^2 - m_\pi^2 c^4)}}{4m_\pi^2 c^4 - 4\hbar^2 y^2 z^2 - t(y,\theta)(1-z)^2})dz\,, \end{aligned} \tag{13}$$

$$A_3(y,\theta)=\frac{\hbar y}{2m_{np}^2c^4}\left(1+2\Delta\mu_p-(1+\Delta\mu_p)^2\cos\theta\right)+$$
$$+\frac{Bt(y,\theta)\,\hbar y}{m_\pi^2c^4-t(y,\theta)}+B\left(\frac{m_\pi^2c^4}{\hbar y}\arcsin^2\left(\frac{\hbar y}{m_\pi c^2}\right)-\hbar y\right)+$$
$$+2B\hbar^4y^4\sin^2\theta\int_0^1dx\int_0^1\frac{x(1-x)\,z(1-z)^3}{W^3(x,y,z,\theta)}Q(x,y,z,\theta)\,dz\,, \tag{14}$$

$$A_4(y,\theta)=-\frac{(1+\Delta\mu_p)^2}{2m_{np}^2c^4\hbar y}+2B\int_0^1dx\int_0^1\frac{z(1-z)}{W(x,y,z,\theta)}\arcsin\frac{\hbar yz}{R(x,y,z,\theta)}dz\,, \tag{15}$$

$$A_5(y,\theta)=\frac{(1+\Delta\mu_p)^2}{2m_{np}^2c^4\hbar y}-\frac{B\hbar y}{m_\pi^2c^4-t(y,\theta)}+B\int_0^1dx\int_0^1(-U(x,y,z,\theta)+L(x,y,z,\theta)\cos\theta)\,dz\,, \tag{16}$$

$$A_6(y,\theta)=\frac{B\hbar y}{m_\pi^2c^4-t(y,\theta)}-\frac{1+\Delta\mu_p}{2m_{np}^2c^4\hbar y}+B\int_0^1dx\int_0^1(U(x,y,z,\theta)-L(x,y,z,\theta))\,dz\,, \tag{17}$$

$$t(y,\theta)=-2\hbar^2y^2(1-\cos\theta)\ , \tag{18}$$

$$W(x,y,z,\theta)=\sqrt{m_\pi^2c^4-\hbar^2y^2z^2+t(y,\theta)(1-z)^2x(x-1)}\,, \tag{19}$$

$$R(x,y,z,\theta)=\sqrt{m_\pi^2c^4+t(y,\theta)(1-z)^2x(x-1)}\,, \tag{20}$$

$$Q(x,y,z,\theta)=\arcsin\frac{\hbar yz}{R(x,y,z,\theta)}+\frac{\hbar yzW(x,y,z,\theta)}{R^2(x,y,z,\theta)}\,, \tag{21}$$

$$U(x,y,z,\theta)=\frac{(1-z)^2}{W(x,y,z,\theta)}\arcsin\frac{\hbar yz}{R(x,y,z,\theta)}\,, \tag{22}$$

$$L(x,y,z,\theta)=2\hbar^2y^2\frac{x(1-x)\,z(1-z)^3}{W^3(x,y,z,\theta)}Q(x,y,z,\theta)\ , \tag{23}$$

$$\alpha=\frac{e^2}{\hbar c}\,, \tag{24}$$

$$B=\frac{g_A^2}{8\pi F_\pi^2}\,. \tag{25}$$

where $\hbar$ is Planck constant, e is elementary charge, $m_\pi c^2=138$ MeV, $m_{np}c^2=938,9$ MeV, g_A =1,257, $F_\pi=92,5$ MeV [4].

Photon energy E_γ in laboratory system (see (7)) is related to y by the equation [4]

$$\hbar y=\frac{E_\gamma}{\sqrt{1+\frac{2E_\gamma}{m_{np}c^2}}}\,. \tag{26}$$

5. Final formula for Baryshevsky–Luboshits effect on protons

Substituting (5) into (4) and regarding (6),(7), we obtain

$$\mathrm{Re}f_p\left(\omega\right) = s\left(\omega, \Delta\mu_p\right) + \frac{\hbar^2\omega\alpha^2 m_{np}c^3}{\pi\left(m_{np}c^2 + 2E_\gamma\right)}\sum_{i=1}^{4}\int_0^{+\infty}\int_0^{\pi}\sin\theta d\theta\frac{yf_i\left(y,\theta\right)dy}{y^2-\omega^2}\,. \tag{27}$$

Regarding (1),(8)–(27), we obtain the final formula for Baryshevsky-Luboshits effect on polarized protons. The final expression is very complicated, it can be structured into terms some of which contain double, triple and quadruple integrals. The integrals can be calculated by numeric methods only.

6. Conclusion

A formula was obtained for Baryshevsky-Luboshits effect on spin-polarized protons depending on the anomalous magnetic moment of a proton and on the difference of Compton scattering cross sections for the same and opposite spin directions of a photon and a proton. The final expression is very complicated, it can be structured into terms some of which contain double, triple and quadruple integrals. The integrals can be calculated by numeric methods only. Further research for neutrons is needed (as well as for quarks).

Acknowledgments

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