Nonlinear Dynamics and Applications

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Proceedings of the Twenty-eight Anniversary Seminar NPCS'2021
in memory of Prof. V.I. Kuvshinov
May 18-21, 2021, Minsk, Belarus
Fractals, Chaos, Phase Transitions, Self-organization

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Preface

The 28th Anniversary International Seminar "Nonlinear Phenomena In Complex Systems" was held in memory of Prof. V.I. Kuvshinov on May 18-21, 2021, in Minsk, Belarus.

The bright memory of

Vyacheslav Ivanovich Kuvshinov

will forever remain in our hearts.

The 28th Anniversary International Seminar traditionally had subsubjects: "Fractals, Chaos, Phase transitions, Self-organization", plenary session and the following section sessions: Particles, Modelling and Safety Related Analyses of NPP, Quantum and Classical Electrodynamics, Gravity, Media, Medicine, Biological and Chemical systems, Mathematics and Fields. 14 plenary, 48 section and 7 poster reports were submitted to the 28th Seminar by the scientists. At this Anniversary Seminar, the participants gave overview reports, which became useful to young scientists. Thus, in addition to the scientific component, the seminar also played an educational role. Most of the papers were included into these Proceedings.

The 28th Anniversary Seminar 'NPCS' was supported by National Academy of Sciences of Belarus.

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To the Problem of the Contribution of Spin-Polarized Hadrons to Baryshevsky—Luboshits Effect at Low Energies of Photons

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Baryshevsky–Podgoretsky effect (nuclear pseudomagnetism) is not the only effect possible on spin polarized nuclei. Another possible effect is similar to Baryshevsky—Luboshits effect on spin polarized electrons. The general structure of the formula for the angle of rotation of the plane of polarization of a photon moving among spin polarized hadrons depends on the difference of Compton forward scattering amplitudes for the same and opposite spin directions of a photon and a hadron. Such a formula depending on the anomalous magnetic moment of a hadron is obtained for low energies of photons. The problem is of interest for astrophysics because spontaneous spin polarization of protons is possible in the outer layers of DA white dwarfs, and spontaneous spin polarization of protons and neutrons is possible at Supernovae II explosions, and if leads to the problem of the contribution of protons and neutrons to Baryshevsky—Luboshits effect in comparison with spin polarized electrons.

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Keywords: Baryshevsky–Luboshits effect, Compton forward scattering amplitudes, spin polarization of protons and neutrons, white dwarfs, Supernovae II explosions.

1. Introduction

Baryshevsky–Luboshits effect is the rotation of photon polarization plane in spin-polarized electron gas explained by the difference of Compton scattering amplitudes for the same and opposite spin directions of photon and electron. This effect was predicted in 1965 and experimentally discovered in 1970s [1].

Another effect which is observed on spin-polarized nuclei (instead of spin-polarized electrons) is Baryshevsky–Podgoretsky effect (nuclear pseudomagnetism) [1] but it is not the only effect possible on spin polarized nuclei. One more effect possible on spin-polarized nuclei is similar to Baryshevsky—Luboshits effect on spin polarized electrons because Compton effect for protons and neutrons (as well as for any other hadrons) is also possible (like for electrons). The approximate formulae for the angle of rotation of photon polarization plane in spin-polarized proton gas were proposed for photon energies large and small in comparison with $m_p c^2$, where m_p is proton mass [2]. It is interesting to obtain the formula for Baryshevsky—Luboshits effect on spin-polarized nucleons

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considering proton and neutron anomalous magnetic moments. The problem can have astrophysical applications because spin polarization of nuclei is predicted in astrophysics also [3].

2. General formula for Baryshevsky–Luboshits effect

If a photon passes the distance dx and its plane of linear polarization turns on the angle $d\varphi$, the following denotations will be used for $\frac{d\varphi}{dx}$ calculation: n_h – particle (hadron) number density ($h = p, n$), n_e — electron number density, $\mathbf{p}_h$ and $\mathbf{p}_e$ — hadron and electron spin polarization vectors, respectively; Ze — hadron charge, e — elementary charge, ω — photon frequency, $\mathbf{n}$ — unit vector in the direction of photon propagation, c — speed of light in vacuum. The general formula for $\frac{d\varphi}{dx}$ calculation in hadron gas is similar to the corresponding formula obtained for electron gas [1]

$$\frac{d\varphi}{dx} = \frac{2\pi n_e c}{\omega} \left(\mathbf{p}_e \cdot \mathbf{n}\right) \mathrm{Re} f_e\left(\omega\right), \tag{1}$$

and can be written in the following way:

$$\frac{d\varphi}{dx} = \frac{2\pi n_h c}{\omega} \left(\mathbf{p}_h \cdot \mathbf{n}\right) \mathrm{Re} f_h\left(\omega\right), \tag{2}$$

where $f_h\left(\omega\right)$ is the spin-dependent part of Compton forward scattering amplitude on a hadron. One can show that it can be expressed as

$$f_h\left(\omega\right) = \frac{f_h^{(+)}\left(\omega\right) - f_h^{(-)}\left(\omega\right)}{2}, \tag{3}$$

where $f_h^{(+)}\left(\omega\right)$ and $f_h^{(-)}\left(\omega\right)$ are Compton scattering amplitudes for the same and opposite directions of spins (helicities) of a photon and a hadron, respectively. According to (2) and (3) we obtain

$$\frac{d\varphi}{dx} = \frac{\pi n_h c}{\omega} \left(\mathbf{p}_h \cdot \mathbf{n}\right) \mathrm{Re}\left(f_h^{(+)}\left(\omega\right) - f_h^{(-)}\left(\omega\right)\right). \tag{4}$$

3. Amplitude difference calculation

The general structure of Compton scattering amplitude on a hadron at $\hbar\omega \ll m_h c^2$ is expressed by Low—Gell-Mann—Goldberger formula [4]

$$M_{fi} = M_{fi}^{(0)} + M_{fi}^{(1)}, \tag{5}$$

$$M_{fi}^{(0)} = -8\pi \left(Ze\right)^2 \left(\mathbf{e}'^{*} \cdot \mathbf{e}\right) \left(w'^{*} w\right), \tag{6}$$

$$M_{fi}^{(1)} = M_{fi}^{(11)} + M_{fi}^{(12)}, \tag{7}$$

$$M_{fi}^{(11)} = -16\pi i \frac{m_h}{\hbar} \mu_{anom(h)}^2 \omega \left(w'^{*} \mathbf{s} w\right) \cdot \left[\left[\mathbf{n}'^{*}, \mathbf{e}'^{*}\right], \left[\mathbf{n}, \mathbf{e}\right]\right], \tag{8}$$

$$M_{fi}^{(12)} = -4\pi i Ze \mu_{anom(h)} \frac{\omega}{c} \left(w'^{*} \mathbf{s} w\right) \cdot \mathbf{l}, \tag{9}$$

$$\begin{aligned} \mathbf{l} = \mathbf{n}\left(\left[\mathbf{n}, \mathbf{e}\right] \cdot \mathbf{e}'^{*}\right) + \left[\mathbf{n}, \mathbf{e}\right]\left(\mathbf{n} \cdot \mathbf{e}'^{*}\right) - \\ -\mathbf{n}'\left(\left[\mathbf{n}', \mathbf{e}'^{*}\right] \cdot \mathbf{e}\right) - \left[\mathbf{n}', \mathbf{e}'^{*}\right]\left(\mathbf{n} \cdot \mathbf{e}\right) - 2\left[\mathbf{e}'^{*}, \mathbf{e}\right] \end{aligned} \tag{10}$$

$$\mathbf{s} = \left(\sigma_x, \sigma_y, \sigma_z\right), \tag{11}$$

where $\mu_{anom(h)}$ is hadron anomalous magnetic moment, $\mathbf{e}$ and $\mathbf{e}'^{*}$ are photon polarization vectors before and after scattering, respectively, $\mathbf{n}'$ is unit vector in the direction of photon propagation after scattering, w and w' are hadron spinors before and after scattering, respectively, $\sigma_x, \sigma_y, \sigma_z$ are Pauli matrices, m_h is hadron mass.

Let's consider a hadron with positive spin projection along z-axis. As far as we consider forward elastic scattering, the corresponding spinors are [4]

$$w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, w'^{*} = \begin{pmatrix} 1 & 0 \end{pmatrix}. \tag{12}$$

As far as photon state doesn't change after scattering, then

$$\mathbf{n} = \mathbf{n}', \tag{13}$$

and for left and right circular polarization one obtains, respectively [1],

$$\mathbf{e}_{-} = \mathbf{e}'_{-} = \frac{1}{\sqrt{2}} \left(\mathbf{e}_1 - i\mathbf{e}_2\right), \tag{14}$$

$$\mathbf{e}_{+} = \mathbf{e}'_{+} = \frac{1}{\sqrt{2}} \left(\mathbf{e}_1 + i\mathbf{e}_2\right), \tag{15}$$

where

$$\mathbf{e}_1 = \left[\mathbf{e}_2, \mathbf{n}\right]. \tag{16}$$

According to (4)-–(15) we obtain

$$M_{fi}^{(0)\pm} = -8\pi \left(Ze\right)^2, \tag{17}$$

$$M_{fi}^{(11)} = \pm 16\pi \frac{m_h}{\hbar} \mu_{anom(h)}^2 \omega, \tag{18}$$

$$M_{fi}^{(12)} = \mp 16\pi Ze\mu_{anom(h)} \frac{\omega}{c}, \tag{19}$$

$$M_{fi}^{+} - M_{fi}^{-} = 32\pi \frac{\mu_{anom(h)}}{\hbar c} \omega \left(m_h c \mu_{anom(h)} - Ze\hbar\right). \tag{20}$$

4. The relationship between M and f amplitudes

As far as for $\omega = \omega'$ the differential cross section $\frac{d\sigma}{d\Omega}$ (where $d\Omega$ is solid angle) [4]

$$\frac{d\sigma}{d\Omega} = \frac{\left|M_{fi}\right|^2}{64\pi^2 m_h^2 c^4} \tag{21}$$

can be also expressed as

$$\frac{d\sigma}{d\Omega} = \left|f\right|^2 \tag{22}$$

it is easy to obtain the relationship

$$f_h^{(\pm)} \left(\omega\right) = \frac{M_{fi}^{(\pm)}}{8\pi m_h c^2}. \tag{23}$$

5. Final formulae for Baryshevsky–Luboshits effect on protons and neutrons

According to (4), (20) and (23) and similar calculations for a hadron with negative spin projection along z-axis it is easy to obtain

$$\frac{d\varphi}{dx} = 4\pi n_h \left(\mathbf{p}_h \cdot \mathbf{n}\right) \frac{\mu_{anom(h)}}{\hbar c} \left(Z \frac{e\hbar}{m_h c} - \mu_{anom(h)} \right). \tag{24}$$

If we consider a photon in spin-polarized proton gas then $h = p$ and

$$\mu_{anom(p)} = \left(\gamma_p - 1\right) \mu_N \approx 1.7928 \mu_N, \tag{25}$$

$$Z \frac{e\hbar}{m_p c} = 2\mu_N. \tag{26}$$

Then, according to (24)–(26) one obtains

$$\frac{d\varphi}{dx} \approx 4\pi n_p \left(\mathbf{p}_p \cdot \mathbf{n}\right) \frac{\mu_N^2}{\hbar c} \cdot 0.3715, \tag{27}$$

where μ_N is nuclear magneton.

If we consider a photon in spin-polarized neutron gas then $h = n$ and

$$\mu_{anom(n)} = -\left(\gamma_n - 1\right) \mu_N \approx 0.9126 \mu_N, \tag{28}$$

$$Z \frac{e\hbar}{m_n c} = 0. \tag{29}$$

Then, according to (24), (28) and (29) one obtains

$$\frac{d\varphi}{dx} \approx -4\pi n_n \left(\mathbf{p}_n \cdot \mathbf{n}\right) \frac{\mu_N^2}{\hbar c} \cdot 0.8328. \tag{30}$$

The results (27) and (30) don't depend on ω. They are similar to the term ($\mu_{anom(e)}$ is electron anomalous magnetic moment)

$$\frac{d\varphi}{dx} \approx 2\pi n_e \left(\mathbf{p}_e \cdot \mathbf{n}\right) \frac{\mu_{anom(e)}^2}{\hbar c} \tag{31}$$

in case of Baryshevsky–Luboshits effect on electrons [1]. This term is usually neglected in calculations because $|\frac{\mu_{anom(e)}}{\mu_B}| \ll 1$ (where μ_B is Bohr magneton). The case of nucleons is different because $|\frac{\mu_{anom(h)}}{\mu_N}| \sim 1$. The next terms in (27) and (30), depending on ω and similar to the next terms in the formula for Baryshevsky–Luboshits effect on electrons, should be considered. This is the task for further research.

6. The results of calculations

Proton spin polarization degree is estimated as $p_p = 0.9$ for temperatures $T = 7 \cdot 10^4$ K and number densities $n_p = n_e = 10^{26}$ cm^{-3} [3]. The expression for magnetic field strength is

$$B = 4\pi p_p n_p \sigma_p \mu_N. \tag{32}$$

For the corresponding values of p_p and n_p one obtains $B = 2.9 \cdot 10^4$ Gs. If we consider electrons to be paramagnetic and non-relativistic, then their spin polarization degree is estimated as [5]

$$p_e = th\left(\frac{\mu_B B}{kT}\right), \tag{33}$$

where μ_B is Bohr magneton.

For the corresponding values of B and T one obtains $p_e \sim 3 \cdot 10^{-5}$.

Comparing (1) to (27) at $n_p = n_e$, one needs to compare $|\frac{c}{\omega} p_e \mathrm{Re} f_e(\omega)|$ to $|2p_p \frac{\mu_N^2}{\hbar c} \cdot 0.3715|$. We can express $\mathrm{Re} f_e(\omega)$ as [1]

$$\mathrm{Re} f_e(\omega) = -\frac{\omega}{2\pi c} r_0^2 \psi(\chi), \chi = \frac{\hbar\omega}{m_e c^2}, \tag{34}$$

$$r_0 = \frac{e^2}{m_e c^2}, \tag{35}$$

where m_e is electron mass, r_0 is electromagnetic electron radius.

If we assume the value of photon energy as 600 keV (such energies are preferable for Baryshevsky–Luboshits effect when ψ has the maximum value of about 0.4 [1]), then, according to (1), (27), (34) and (1), we obtain

$$|\frac{r_0^2}{2\pi} p_e \psi(\chi)| \sim 2\alpha \cdot 10^{-29}\mathrm{cm}^2, |2p_p \frac{\mu_N^2}{\hbar c} \cdot 0.3715| \sim 7\alpha \cdot 10^{-29}\mathrm{cm}^2, \tag{36}$$

where α is electromagnetic coupling constant.

It's obvious that the values in (36) are of the same order of magnitude. As far as $p_p \uparrow\downarrow p_e$ in magnetic field, then, considering the sign of $\mathrm{Re} f_e(\omega)$ in (34), one comes to the conclusion that the absolute values of $\frac{d\varphi}{dx}$ in (1) and (27) are to be added in order to find the total contribution of spin-polarized electrons and protons to Baryshevsky—Luboshits effect in the outer layers of white dwarfs. The result of the calculations is $\frac{d\varphi}{dx} \sim 4 \cdot 10^{\breve{}4}$ rad/cm at $n_p = n_e = 10^{26}$ cm^{-3}. This value is one order lower than the corresponding value for electrons in iron where $p_e \approx 7.85 \cdot 10^{-2}, n_e \approx 8.5 \cdot 10^{26}$ cm^{-3} [1].

It's obvious that the right parts of (27) and (30) are of the same order of magnitude if $n_p \sim n_n$ and $|p_p| \sim |p_n|$. But if we assume $n_n = 10^{26}$ cm^{-3} then $p_n \sim 3 \cdot 10^{-5}$ at the conditions mentioned above [3]. Comparing (30) to (27) it's easy to estimate that the contribution of spin-polarized neutrons to Baryshevsky—Luboshits effect can be neglected in comparison with the contribution of electrons and protons at Supernovae II explosions.

7. Summary

A formula for Baryshevsky–Luboshits effect on spin-polarized hadrons depending on the anomalous magnetic moment of a hadron was obtained for low energies of photons. The contribution of spin-polarized protons to Baryshevsky–Luboshits effect in comparison with that of spin-polarized electrons in the outer layers of DA white dwarfs was estimated. The contribution of spin-polarized neutrons to Baryshevsky–Luboshits effect in comparison with that of spin-polarized protons at Supernovae II explosions was estimated.

It was shown that

- the contribution of spin-polarized protons to Baryshevsky–Luboshits effect in comparison with that of spin-polarized electrons is of the same order in 10^2 keV range when proton spin polarization degree is five orders higher than that of electrons;

- the contribution of spin-polarized neutrons to Baryshevsky–Luboshits effect in comparison with that of spin-polarized protons is of the same order at low energies of photons if the number densities and spin polarization degrees of neutrons and protons are of the same order of magnitude;
- further terms depending on ω should be considered.

8. Acknowledgement

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