Groups with $\mathfrak{A}^2$-subnormal subgroups

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We consider only finite groups. All notations and terminology are standard [1]. By $\mathfrak{N}$, $\mathfrak{A}$ and $\mathfrak{E}$ we denote the class of all abelian, nilpotent and all groups, respectively; $F(G)$ denotes the Fitting subgroup of a group G.

Let $\mathfrak{F}$ be a formation, G be a group. The subgroup $G^{\mathfrak{F}} = \bigcap\{N \lhd G : G/N \in \mathfrak{F}\}$ is the smallest normal subgroup of G with quotient in $\mathfrak{F}$, and it is called the $\mathfrak{F}$-residual of G. A subgroup H of a group G is called $\mathfrak{F}$-subnormal if there is a chain of subgroups

$$H = H_0 <\cdot\, H_1 <\cdot\, \ldots <\cdot\, H_n = G$$

such that $H_i/(H_{i-1})_{H_i} \in \mathfrak{F}$ for all i, that is equivalent to $H_i^{\mathfrak{F}} \leqslant (H_{i-1})_{H_i}$. Here $Y_X = \bigcap_{x \in X} Y^x$ denotes the core of Y in X, $H_{i-1} <\cdot\, H_i$ denotes that H_{i-1} is a maximal subgroup of H_i.

If $\mathfrak{X}$ and $\mathfrak{F}$ are s-closed formations, then the product

$$\mathfrak{X}\mathfrak{F} = \{\, G \in \mathfrak{E} \mid G^{\mathfrak{F}} \in \mathfrak{X}\},$$

by [1, p. 337], is an s-closed formation. When $\mathfrak{X} = \mathfrak{F}$, we write $\mathfrak{X}^2$ instead of $\mathfrak{X}\mathfrak{F}$.

Groups with various collections of $\mathfrak{F}$-subnormal subgroups are investigated by many authors, see references of [2–4].

It is easy to prove that every Sylow subgroup of any soluble group is $\mathfrak{A}\mathfrak{N}$-subnormal. Therefore in the universe of all soluble groups the class of groups with $\mathfrak{F}$-subnormal Sylow subgroups should be investigated when $\mathfrak{F}$ does not contain $\mathfrak{A}\mathfrak{N}$.

Theorem. *In a group G every Sylow subgroup is $\mathfrak{A}^2$-subnormal in G if and only if G is soluble and every Sylow subgroup of $G/F(G)$ is abelian.*

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