

# To the Problem of Compton Rotation of Photons in a Strong Magnetic Field: Limit of Total Spin Polarization of Electrons

A. I. Sery\*

*A. S. Pushkin Brest State University, Brest, BELARUS*

(Received 05 September, 2014)

A formula is obtained for Compton rotation angle of the plane of linear polarization of photons per unit path in electron gas with high degree of spin polarization of electrons in quantizing magnetic field.

**PACS numbers:** 05.30.Fk, 41.50.+h, 42.25.Ja, 71.70.Di, 78.20.Ek

**Keywords:** Compton rotation, electron gas, quantizing magnetic field

## 1. Introduction

The effect of Compton rotation of the plane of polarization of X- and gamma-photons in the absence of a magnetic field was predicted by V.G. Baryshevsky and V.L. Luboshitz in 1965 and experimentally tested on iron at early 1970s; the degree of electron spin polarization did not exceed 8% [1]. At the presence of a strong magnetic field (at a neutron star surface) the spin polarization is almost total, electron wave functions are changed. That is why we need to recalculate the difference of Compton scattering forward amplitudes for right and left circular polarizations of a photon. The method applied here was developed in [2].

Let us consider a constant homogeneous magnetic field with a strength  $B$  directed along  $z$  axis. Let us choose the following gauge of the vector potential:

$$A_0 = A_x = A_z = 0, A_y = Bx. \quad (1)$$

At elastic forward scattering, the 4-momentum of initial and final photon (moving at angle  $\theta$  to  $z$  axis) with a wave vector  $k$  and frequency  $\omega$  do not change:

$$\begin{aligned} ck_0 &= \omega, & ck_x &= 0, \\ ck_y &= \omega \sin \theta, & ck_z &= \omega \cos \theta. \end{aligned} \quad (2)$$

A photon with linear polarization is a superposition of 2 circularly polarized photons with opposite helicities. The 4-vector of right (+) or left (-) circular photon polarization is (transposing is denoted by  $T$ ):

$$\sqrt{2}e_{(\pm)}^T = [0 \mp i \cos \theta \ -\sin \theta]. \quad (3)$$

The 4-momentum of a virtual electron in  $R$ -process ( $g$ ) and  $S$ -process ( $f$ ) is

$$\begin{aligned} c\lambda_0^\pm &= \varepsilon_0 \pm \hbar\omega, & c\lambda_2^\pm &= p_y c \pm \hbar\omega \sin \theta, \\ c\lambda_3^\pm &= p_z c \pm \hbar\omega \cos \theta, & \lambda^+ &= g, \lambda^- = f. \end{aligned} \quad (4)$$

If  $e$  is the electric charge,  $m, p_y, p_z$  are electron's mass and momentum along  $y, z$  axes, respectively, then one can write out the electron wave functions using the bispinors  $u_n$  and Hermitian functions  $H_n$ ,  $n \geq 0$ :

$$\begin{aligned} \Psi_0(x) &= iA_0\sqrt{2eB\hbar}u_0, \\ U_n(x) &= \frac{1}{\sqrt{2}}\exp(-\frac{x^2}{2})H_n(x), \\ \varepsilon_0 &= \sqrt{m^2c^4 + p_z^2c^2}, \\ A_0^{-1} &= \sqrt{2c\hbar\varepsilon_0(\varepsilon_0 + mc^2)}\sqrt{Be\hbar}, \\ u_0^T &= [0 \ -mc^2 - \varepsilon_0 \ 0 \ p_z c]. \end{aligned}$$

Dirac matrices  $\gamma_k$  ( $k = 0, 1, 2, 3$ ) are written in standard presentation. Electron propagator

\*E-mail: alexey\_sery@mail.ru

contains structures of the following type – ( $\varepsilon_{n\lambda}$  is the energy of virtual electron on intermediate  $n$ -th Landau level,  $\Gamma_n$  is the level width):

$$\begin{aligned} G_B(\lambda, x) &= \sqrt{\frac{Be}{c\hbar}} \sum_{n=0}^{\infty} \frac{\hbar c^2}{c^2 \lambda_0^2 - \varepsilon_{n\lambda}^2 - i \cdot 0} D_n(\lambda), Y(\lambda) = \gamma_0 \lambda_0 - \gamma_3 \lambda_3 + mc, \\ D_n(\lambda) &= U_n(x_1) U_n(x_2) Y(\lambda) \beta_1 + (1 - \delta_{0n}) U_{n-1}(x_1) U_{n-1}(x_2) Y(\lambda) \beta_2 \\ &+ (1 - \delta_{0n}) i \sqrt{\frac{2neB\hbar}{c}} (U_{n-1}(x_1) U_n(x_2) \gamma_1 \beta_1 - U_n(x_1) U_{n-1}(x_2) \beta_1 \gamma_1), \\ 2\beta_1 &= 1 + i\gamma_2 \gamma_1, 2\beta_2 = 1 - i\gamma_2 \gamma_1, \quad \varepsilon_{n\lambda} = \sqrt{m^2 c^4 + 2ne\hbar Bc + \lambda_3^2 c^2} - \frac{i\Gamma_n}{2}. \end{aligned}$$

## 2. Calculation of the rotation angle

If  $k_x = 0$  then the matrix element of the process is [2]:

$$\begin{aligned} \Omega &= \frac{-i\alpha(2\pi\hbar)^4 c e_\mu e'_\nu{}^*}{\hbar V S_0 \omega} \delta^3(\mathbf{p} + \hbar\mathbf{k} - \mathbf{p}' - \hbar\mathbf{k}') \chi_{\mu\nu}, \quad \alpha = \frac{e^2}{\hbar c}, \\ \chi_{\mu\nu} &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx_1 dx_2 \bar{\Psi}_0(\xi_1) (\gamma_\nu G_B(g, \rho) \gamma_\mu + \gamma_\mu G_B(f, \eta) \gamma_\nu) \Psi_0(\xi_2), \end{aligned} \quad (5)$$

$$j_r(p) = \sqrt{\frac{eB}{c\hbar}} (x_r + \frac{cp}{eB}), \quad \xi_r = j_r(p_y), \quad \rho_r = j_r(g_2), \eta_r = j_r(f_2), \quad r = 1, 2. \quad (6)$$

Here  $V$  is the normalization volume for the photon,  $S_0$  is the normalization area for the electron in  $(xy)$  plane.  $\delta$ -function includes  $y$ -,  $z$ - and  $t$ -components. The formula for the Compton rotation angle of the plane of linear polarization of hard X-photon per unit path ( $n_e$  is the electron density) is [1]:

$$\frac{d\varphi}{dl} = \frac{\pi n_e c}{\omega} \Re(F_{(-)} - F_{(+)}). \quad (7)$$

Here  $F_{\pm}$  are Compton forward scattering amplitudes for right (+) and left (–) circular photon polarizations. The relationship between

$F$  in (7) and  $\Omega$  in (5), according to [1] and [3] (making a substitution for the  $x$ -component of  $\delta$ -function through  $V$ ,  $S_0$ ), is:

$$\begin{aligned} \Omega &= (\varepsilon_0 + \hbar\omega) \\ &\times \frac{i(2\pi\hbar)^4 c F}{V S_0 \varepsilon_0 \hbar \omega} \delta^3(\vec{p} + \hbar\vec{k} - \vec{p}' - \hbar\vec{k}'). \end{aligned} \quad (8)$$

Subject to the expressions (5), (6), and (8) we obtain for Eq. (7) the following formula ( $\mu_B$  is the Bohr magneton):

$$\begin{aligned}
\frac{d\varphi}{dl} &= \frac{(\pi\hbar c)^2 n_e \alpha \cos \theta}{\hbar\omega(\varepsilon_0 + \hbar\omega)} \exp\left(-\frac{\phi}{2}\right) \sum_{n=1}^{\infty} \phi^{n-1} \Re(\Xi_n^{(+)}(g, \theta) - \Xi_n^{(-)}(f, \theta)), \\
\phi &= \frac{\hbar\omega^2 \sin^2 \theta}{cBe}, \quad \Lambda_n(\lambda) = c^2 \lambda_0^2 - \varepsilon_{n\lambda}^2 + \frac{\Gamma_n^2}{4}, \quad \Gamma_n \approx \frac{16(2n-1)\alpha(\mu_B B)^2}{3mc^2}, \\
\Xi_n^{(\pm)}(\lambda, \theta) &= \frac{((c\lambda_0 \varepsilon_0 - m^2 c^4) \cos \theta - p_z c(c\lambda_3 \cos \theta \pm \sqrt{2n} \hbar\omega \sin^2 \theta)) \Lambda_n(\lambda)}{\Lambda_n^2(\lambda) + \Gamma_n^2 \varepsilon_{n\lambda}^2}.
\end{aligned} \tag{9}$$

For example, if  $n_e \sim 10^{23} \text{ cm}^{-3}$ ,  $\hbar\omega \sim 0.1 \text{ MeV}$ ,  $B \sim 10^{13} \text{ Gs}$ , then  $d\varphi/dl \sim 10^2 - 10^3 \text{ rad/cm}$  at the resonance on  $n$ -th Landau level of virtual electron.

### 3. Summary

The difference of the Compton forward scattering amplitudes in quantizing magnetic field is calculated for an electron in the ground state and circularly polarized X-photons moving at an arbitrary angle to magnetic field strength lines

with opposite helicities. A corresponding formula is obtained for calculation of the Compton rotation angle of the plane of linear polarization of photons per unit path in electron gas at the limit of total spin polarization of electrons.

### Acknowledgement

In present work the research is done according to the suggestion of V. G. Baryshevsky and V. V. Tikhomirov.

### References

- [1] V.G. Baryshevskii. *Nuclear Optics of Polarized Media* (Energoatomizdat, Moscow, 1995). (In Russian).
- [2] P.I. Fomin, R.I. Kholodov. JETP. **90**, no. 2, 281 (2000).
- [3] V.B. Berestetskii, E.M. Lifshitz and L.P. Pitaevskii. *Quantum Electrodynamics (Course of theoretical physics, Vol 4, 2nd ed.)* [transl. from the Russian by J.B. Sykes and J.S. Bell] (Pergamon Press, 1982).