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IRREGULARIZABILITY OF THE RIEMANN – HILBERT BOUNDARY VALUE PROBLEM FOR ONE BIHARMONIC SYSTEM IN $\mathbb{R}^3$

Let Ω be a simply connected bounded, homeomorphic to a ball, domain in $\mathbb{R}^3$ with a sufficiently smooth boundary $\partial\Omega$. We consider in Ω the system of four first-order differential equations of the form

$$\frac{\partial U}{\partial x_1} + \begin{pmatrix} 0 & -1 & 2 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \frac{\partial U}{\partial x_2} + \begin{pmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \end{pmatrix} \frac{\partial U}{\partial x_3} = 0, \qquad (1)$$

where $U = (u_1(x),\, u_2(x),\, u_3(x),\, u_4(x))^T$ is a column vector of the real-valued functions, T is for transpose and $x = (x_1,\, x_2,\, x_3) \in \mathbb{R}^3$.

We note that the system (1) is not a three-dimensional analog of the Cauchy – Riemann system [1]. However, it is elliptic and each component of an arbitrary continuously differentiable solution of the system (1) satisfies the biharmonic equation [2] in $\mathbb{R}^3$. Such systems are classified to the systems of the biharmonic type [3].

We consider the Riemann – Hilbert boundary value problem of finding a solution of the system (1), satisfying the boundary conditions

$$u_1|_{\partial\Omega} = f_1(y), \quad (u_2\nu_1 + u_3\nu_2 + u_4\nu_3)|_{\partial\Omega} = f_2(y) \quad (y \in \partial\Omega), \qquad (2)$$

where $\nu(y) = (\nu_1(y), \nu_2(y), \nu_3(y))$ is the unit field of internal normals on the surface $\partial\Omega$, f is the given Hölder continuous on the surface $\partial\Omega$ the two-component vector-function.

Theorem. [4] *The Riemann-Hilbert problem for system* (1) *and boundary condition* (2) *is not regularisable.*

To prove the theorem, it is shown that at the point of the surface $\partial\Omega$, in which the internal normal is parallel to the axis Ox_1, rank of the Lopatinskiy matrix [5] of the problem (1), (2) is not maximal.

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