Corrections to the Formula for Baryshevsky-Luboshitz Effect in Magnetic Field

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Abstract

In the framework of tree approximation a correction is obtained to the formula for Baryshevsky-Lubositz rotation of the plane of linear polarization of a photon in electron gas with high degree of spin polarization of electrons in magnetic field. The frequency of photon is considered to be of the same order as the cyclotron frequency.

1 Introduction

The effect of rotation of the plane of polarization of X- and gamma-photons on spin-polarized electrons was theoretically predicted by V.G. Baryshevsky and V.L. Luboshitz in 1965 and experimentally tested at early 1970s [1, 2, 3, 4]. The effect was considered for the case when the frequency of photon was much greater than the cyclotron frequency. The effect is possible due to the dependence of Compton scattering forward amplitudes on the relative direction of spins of photon and electron. The effect is important in studying white dwarfs and neutron stars, namely, their magnetic fields and the structures of their atmospheres.

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2 The contribution of Faraday effect

Baryshevsky-Lubositz effect differs from another type of rotation of the plane of polarization of photons known as Faraday effect. The main differences between 2 effects are presented in Table 1.

Table 1 - The difference between Faraday and Baryshevsky-Lubositz effect.

Effect	Faraday	Baryshevsky-Lubositz
Based on	Zeeman effect	the dependence of Compton scattering forward amplitude on the directions of electron and photon spins
Spectral region	radio and visible	hard X and gamma
Can electrons be regarded as free	no	yes
Is spin polarization of electrons necessary	no	yes

The conditions for Faraday effect change significantly in the atmospheres of white dwarfs and neutron stars in comparison with terrestrial conditions because the atomic structure of matter can be destroyed by strong magnetic fields. The meaning of the term "Faraday effect" also changes (see Table 2 for details).

Table 2 - Different variants of Faraday effect.

Variant	Classic	Non-classic
1. Atomic structure	exists	doesn't exist
2a. Electron energy levels are	discrete	discrete-continuous
2b. Quantizing	Bohr-like	Landau
3. Spin degrees of freedom	are not involved	are not involved
4. Ionization at $B \ll 10^9$ Gs	is to be low	is to be high
5a. Can the effect take place at $B \geq 10^9$ Gs	no because condition 1 is not fulfilled	yes
5b. That's why at $B \geq 10^9$ Gs Baryshevsky-Luboshitz effect	is the only type of rotation	exists together with Faraday effect

In non-classic case, it's hard to consider 2 effects separately at $\hbar\omega \approx 2\mu_B B$, but Baryshevsky-Luboshitz effect dominates far from resonances (see also Table 3). The general meaning of the term "Faraday effect" includes both classic and non-classic cases.

Table 3 - Baryshevsky-Lubositz effect at different conditions.

Photon energy	$\hbar\omega \gg 2\mu_B B$	$\hbar\omega \approx 2\mu_B B$
The influence of magnetic field on the effect	can be neglected	is considerable
Spin polarization of electrons is	less then 8% in iron (experiments of 1970s)	expected to be almost 100% in astrophysics in strong magnetic fields
The order of perturbation theory on $e^2/(\hbar c)$	2	1

Baryshevsky-Lubositz effect has also some similar aspects with Baryshevsky-Podgoretsky effect [1] (see Table 4 for details).

Table 4 - Baryshevsky-Lubositz and Baryshevsky-Podgoretsky effects.

Effect	Baryshevsky-Lubositz	Baryshevsky-Podgoretsky
Particle	photon	neutron
Moving	in spin-polarized electron gas	among spin-polarized nuclei
Is spin polarization necessary	yes	yes
What takes place	rotation of the plane of linear polarization of the photon	spin precession of the neutron
Based on	the dependence of Compton scattering forward amplitude on the directions of electron and photon spins	the dependence of scattering forward amplitude on the directions of neutron and nuclear spins
Interaction	electromagnetic	strong (nuclear)
At resonances the value of	rotation changes its sign	precession changes its sign

3 General formula

In [5, 6], using the approach of [7], a formula was obtained for the calculation of Baryshevsky-Lubositz rotation angle of the plane of linear polarization of photons per unit path in electron gas with total spin polarization of electrons ($p_{0e} = 1$). After some simple rearrangements it can be written in the form:

$$\frac{d\varphi}{dl} = \frac{\pi n_e c \alpha \varepsilon_0}{\omega(\varepsilon_0 + \hbar\omega)}(E^{(+)} - E^{(-)}) Re \left(\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx_1 dx_2 \bar{\Psi}_0(\xi_1) Q_{\mu\nu} \Psi_0(\xi_2) \right), \quad (1)$$

where

$$E^{(\pm)} = e_\mu^{(\pm)} e_\nu^{\prime(\pm)*}, \varepsilon_0^2 = m^2c^4 + p_z^2c^2, e^{(\pm)} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & \mp i & \cos\theta & -\sin\theta \end{bmatrix}^T,$$

$$\Psi_0(x) = \frac{i\sqrt{Be}}{\sqrt{2\varepsilon_0(\varepsilon_0 + mc^2)\sqrt{Bec\hbar}}} \exp(-\frac{x^2}{2}) \begin{bmatrix} 0 & -mc^2 - \varepsilon_0 & 0 & p_z c \end{bmatrix}^T,$$

$$\alpha = \frac{e^2}{\hbar c}, j_k(p) = \sqrt{\frac{eB}{\hbar c}}(x_k + \frac{cp}{eB}), \xi_k = j_k(p_y), \rho_k = j_k(g_2), \eta_k = j_k(f_2),$$

$$Q_{\mu\nu} = \gamma_\nu G_B(g,\rho)\gamma_\mu + \gamma_\mu G_B(f,\eta)\gamma_\nu, \beta_1 = \frac{1}{2}(1 + i\gamma_2\gamma_1), \beta_2 = \frac{1}{2}(1 - i\gamma_2\gamma_1),$$

$$cg_0 = \varepsilon_0 + \hbar\omega, cg_3 = p_z c + \hbar\omega\cos\theta, cf_0 = \varepsilon_0 - \hbar\omega, cf_3 = p_z c - \hbar\omega\cos\theta,$$

$$Y = \gamma_0\lambda_0 - \gamma_3\lambda_3 + mc, G_B(\lambda, x) = \sqrt{\frac{Be}{c\hbar}} \sum_{n=0}^{\infty} \frac{\hbar c^2}{c^2\lambda_0^2 - \varepsilon_{n\lambda}^2} D,$$

$$D = U_n(x_1)U_n(x_2)Y\beta_1 + (1 - \delta_{0n})U_{n-1}(x_1)U_{n-1}(x_2)Y\beta_2 +$$

$$+(1 - \delta_{0n})i\sqrt{\frac{2neB\hbar}{c}}(U_{n-1}(x_1)U_n(x_2)\gamma_1\beta_1 - U_n(x_1)U_{n-1}(x_2)\beta_1\gamma_1),$$

$$\varepsilon_{ng} \approx \sqrt{m^2c^4 + 2ne\hbar Bc + g_3^2c^2} - i\frac{8(2n-1)\alpha(\mu_B B)^2}{3mc^2},$$

$$\varepsilon_{nf} \approx \sqrt{m^2c^4 + 2ne\hbar Bc + f_3^2c^2} \quad (2)$$

Here n_e is electron density, m, p_z are electron's mass and momentum along z axis, respectively; μ_B is Bohr magneton, e is electric charge, $\hbar\omega$ is photon's energy, $\vec{B}$ is magnetic field strength, θ is the angle between the wave vector of photon $\vec{k}$ and $\vec{B}$. Transposing is denoted by T. Dirac matrices γ_k ($k = 0, 1, 2, 3$) are in standard presentation. $\varepsilon_{n\lambda}$ is energy of virtual electron on intermediate nth Landau level.

4 Summation over μ, ν

Nonzero contributions in (1) correspond to 2 cases: 1) $\mu = 1, \nu = 2$ and $\mu = 2, \nu = 1$; 2) $\mu = 1, \nu = 3$ and $\mu = 3, \nu = 1$. Only the first case was considered in [6] with the following result:

$$\frac{d\varphi}{dl} = \frac{(\pi\hbar c)^2 n_e \alpha \cos\theta}{\hbar\omega(\varepsilon_0 + \hbar\omega)} \exp\left(-\frac{\phi}{2}\right) \sum_{n=1}^{\infty} \phi^{n-1} Re(\Xi_n(g) - \Xi_n(f)),$$
$$\phi = \frac{\hbar\omega^2 \sin^2\theta}{cBe}, \Xi_n(\lambda) = \frac{c\lambda_0\varepsilon_0 - \lambda_3 p_z c^2 - m^2c^4}{c^2\lambda_0^2 - \varepsilon_{n\lambda}^2}. \quad (3)$$

Considering both cases, one obtains:

$$\frac{d\varphi}{dl} = \frac{(\pi\hbar c)^2 n_e \alpha}{\hbar\omega(\varepsilon_0 + \hbar\omega)} \exp\left(-\frac{\phi}{2}\right) \sum_{n=1}^{\infty} \phi^{n-1} Re(\Xi_n^{(+)}(g,\theta) - \Xi_n^{(-)}(f,\theta)),$$
$$\Xi_n^{(\pm)}(\lambda,\theta) = \frac{(c\lambda_0\varepsilon_0 - m^2c^4)\cos\theta - p_z c(c\lambda_3\cos\theta \pm \sqrt{2n}\hbar\omega\sin^2\theta)}{c^2\lambda_0^2 - \varepsilon_{n\lambda}^2}. \quad (4)$$

The numerical results for (3) and (4) coincide at $p_z = 0$ approximation.

5 Averaging over momenta at T=0 K

The result (3) was averaged over electron momenta p_z at T=0 K in [6]. The same averaging of (4) gives:

$$\frac{d\varphi}{dl} = \frac{e^2 m\mu_B B}{4\hbar^3\omega} \exp\left(-\frac{\phi}{2}\right) \sum_{n=1}^{\infty} \phi^{n-1}(R_n - S_n),$$
$$R_n = \int_{-w_1}^{w_1} \frac{f_1(w)\left(f_2(w)\cos\theta - 2\sqrt{2n}w\sin^2\theta\right)dw}{f_3(w)\left(f_1^2(w) + \frac{\Gamma_n^2}{\hbar^2\omega^2}\left(1 + 4n\frac{\mu_B B}{mc^2} + (w + tcos\theta)^2\right)\right)},$$
$$S_n = \int_{-w_1}^{w_1} \frac{\left(-f_2(w)\cos\theta + 2\sqrt{2n}w\sin^2\theta\right)dw}{f_3(w)\left(Q_n - f_2(w)\right)}, Q_n = tsin^2\theta - 4n\frac{\mu_B B}{\hbar\omega},$$

$$f_1(w) = Q_n + f_2(w) + \frac{\Gamma_n^2}{4\hbar\omega mc^2}, f_2(w) = 2(\sqrt{1+w^2} - w cos\theta),$$
$$f_3(w) = \sqrt{1+w^2} + t, t = \frac{\hbar\omega}{mc^2}, w = \frac{p_z}{mc}, w_1 = \frac{\pi^2(\hbar c)^3 n_e}{(mc^2)^2\mu_B B}. \quad (5)$$

Similarly to [6], the integrals can be taken numerically or analytically. The following notations will be used:

$$\xi_n = -\frac{4cos\theta}{2+Q_n}, q_n = \frac{2-Q_n}{2+Q_n}, \nu_n = 4y_1^2 - \xi_n^2, \mu_n = 4q_n - \xi_n^2,$$
$$\tau_{n\pm} = \xi_n \pm 2y_1, y_1 = \frac{w_1 + \sqrt{1+w_1^2} - 1}{w_1 + \sqrt{1+w_1^2} + 1},$$
$$Y_n = \left(\arctan\left(\frac{\tau_{n+}}{\sqrt{|\mu_n|}}\right) - \arctan\left(\frac{\tau_{n-}}{\sqrt{|\mu_n|}}\right)\right)\tilde{\theta}(\mu_n) +$$
$$+\ln\left|\frac{\left(2y_1 - \sqrt{|\mu_n|}\right)^2 - \xi_n^2}{\left(2y_1 + \sqrt{|\mu_n|}\right)^2 - \xi_n^2}\right|\tilde{\theta}(-\mu_n) \quad (6)$$

Here $\tilde{\theta}(\eta)$ is Heaviside function. Then for S_n-terms one obtains (analytical expressions for R_n-terms are very complicated):

$$S_n = \frac{\sqrt{2n}}{\cos\theta}\left(\tilde{I}_{1n}\sin^2\theta + 2\tilde{I}_{2n} - (Q_n + 2t)\, I_n \sin^2\theta\right) +$$
$$+\left(\tilde{I}_{1n} - Q_n I_n\right)\cos\theta,$$
$$\tilde{I}_{1n} = 2\ln\left(w_1 + \sqrt{w_1^2+1}\right) - \frac{4t}{\sqrt{1-t^2}}\arctan\left(y_1\sqrt{\frac{1-t}{1+t}}\right),$$
$$\tilde{I}_{2n} = -\frac{1+\cos^2\theta}{2\cos\theta}\ln\left|\frac{1-y_1 cos\theta}{1+y_1 cos\theta}\right| - \ln\left|\frac{1+y_1}{1-y_1}\right|, Q_n = -2;$$
$$\tilde{I}_{2n} = \frac{8y_1}{\nu_n}\left(\frac{2\sin^2\theta}{2+Q_n} + q_n\right) - \ln\left|\frac{1+y_1}{1-y_1}\right|$$
$$-\frac{4\xi_n y_1}{\nu_n}\cos\theta - \ln\left|\frac{\tau_{n+}}{\tau_{n-}}\right|\cos\theta, Q_n \neq -2, \mu_n = 0;$$
$$\tilde{I}_{2n} = \left(\xi_n\cos\theta - 2\left(\frac{2\sin^2\theta}{2+Q_n} + q_n\right)\right)\frac{Y_n}{\sqrt{|\mu_n|}} - \ln\left|\frac{1+y_1}{1-y_1}\right| -$$
$$-\frac{1}{2}\ln\left|\frac{y_1^2 + \xi_n y_1 + q_n}{y_1^2 - \xi_n y_1 + q_n}\right|\cos\theta, Q_n \neq -2, \mu_n \neq 0. \quad (7)$$

The expressions for I_n were presented in [6].

6 Numerical results

Some numerical results are compared in Table 5.

Table 5 - The angle of rotation calculated: I) according to (3) and (4) in $p_z = 0$ approximation; II) according to (5) at $n_e = 10^{22}$ cm^{-3}.

	$B = 10^{13}$ Gs			$B = 4 \cdot 10^{13}$ Gs		
θ, deg	$\hbar\omega$, MeV	I	II	$\hbar\omega$, MeV	I	II
30	0.1125	-609.8	-609.4	0.4168	-16.3	-16.2
45	0.1097	-490.4	-490.3	0.3864	-13.4	-13.3
60	0.1072	-342.1	-342.0	0.3629	-9.5	-9.4

The difference between the result for (5) and the corresponding result in [6] is less than 10^{-10} rad/cm, i.e. much less than the accuracy of the results obtained in the first order of perturbation theory on α.

7 Summary. The main results

In the framework of tree approximation a correction is obtained to the formula for Baryshevsky-Lubositz rotation of the plane of linear polarization of a photon in electron gas with high degree of spin polarization of electrons in magnetic field. The frequency of photon is considered to be of the same order as the cyclotron frequency. The numerical difference between the $p_z = 0$ approximation and the averaging on p_z is small. The numerical contribution of $\mu = 1, \nu = 3$ and $\mu = 3, \nu = 1$ is negligibly small in comparison with the contribution of $\mu = 1, \nu = 2$ and $\mu = 2, \nu = 1$.
The research was done according to the suggestion of V.G. Baryshevsky and V.V. Tikhomirov.

References

[1] V.G. Baryshevskii.*Nuclear Optics of Polarized Media* [in Russian] (Energoatomizdat, Moscow, 1995).

[2] V.M. Lobashov *et al.*,[in Russian] Sov. JETP Lett. **14** (1971) 373.

[3] V.M. Lobashov *et al.*,[in Russian] Sov. JETP **68** (1975) 1220.

[4] P. Bock and P. Luksch, Lett. Nuovo cimento **2** (1972) 1081.

[5] A.I. Sery. To the Problem of Compton Rotation of the plane of Polarization of X-Photons in Magnetic Field // Actual Problems of Microworld Physics: Proceedings of International School-Seminar, Gomel, Belarus, August 1 – 12, 2011, (Dubna, 2013) 230.

[6] A.I. Sery. Resonance Width Consideration for Compton Rotation in Magnetic Field // Actual Problems of Microworld Physics: Proceedings of XII International School-Seminar, Gomel, Belarus, July 22 – August 2, 2013, (Dubna, 2015) 194.

[7] P.I. Fomin and R.I. Kholodov, JETP. **90**, 2 (2000) 281.