Problems of Physics, Mathematics and Technics, № 3 (40), 2019

УДК 512.542

КОНЕЧНЫЕ ГРУППЫ С ОГРАНИЧЕНИЯМИ НА ДВЕ МАКСИМАЛЬНЫЕ ПОДГРУППЫ

В.С. Монахов, А.А. Трофимук, Е.В. Зубей

Гомельский государственный университет им. Ф. Скорины

FINITE GROUPS WITH RESTRICTIONS ON TWO MAXIMAL SUBGROUPS

V.S. Monakhov, A.A. Trofimuk, E.V. Zubei

F. Scorina Gomel State University

Подгруппа A называется *полунормальной* в группе G, если существует подгруппа B такая, что $G = AB$ и AB_1 – собственная в G подгруппа для каждой собственной подгруппы B_1 из B. Если подгруппа A либо субнормальна в G, либо полунормальна в G, то A называется *полусубнормальной* в группе G. В настоящей работе доказана сверхразрешимость группы G при условии, что все силовские подгруппы из двух несопряженных максимальных подгрупп полусубнормальны в группе G. Установлена нильпотентность второго коммутанта $(G')'$ группы G при условии, что все максимальные подгруппы из двух несопряженных максимальных подгрупп полусубнормальны в группе G.

Ключевые слова: *сверхразрешимая группа*, *полусубнормальная подгруппа*, *коммутант*, *силовская подгруппа*, *максимальная подгруппа*.

A subgroup A of a group G is called *seminormal* in G, if there exists a subgroup B such that $G = AB$ and AB_1 is a proper subgroup of G for every proper subgroup B_1 of B. We introduce the new concept that unites subnormality and seminormality. A subgroup A of a group G is called *semisubnormal* in G, if either A is subnormal in G, or is seminormal in G. In this paper we proved the supersolubility of a group G under the condition that all Sylow subgroups of two non-conjugate maximal subgroups of G are semisubnormal in G. Also we obtained the nilpotency of the second derived subgroup $(G')'$ of a group G under the condition that all maximal subgroups of two non-conjugate maximal subgroups are semisubnormal in G.

Keywords: *supersoluble groups*, *semisubnormal subgroup*, *derived subgroup*, *Sylow subgroup*, *maximal subgroup*.

Introduction

Throughout this paper, all groups are finite and G always denotes a finite group.

A subgroup A of a group G is called *seminormal* in G, if there exists a subgroup B such that $G = AB$ and $AB_1 = B_1A \neq G$ for every proper subgroup B_1 of B.

Groups with some seminormal subgroups were investigated in works of many authors, see, for example, [1]–[10]. In particular, the supersolubility of a group with seminormal Sylow subgroups was obtained in [7], [9]. In [6] the supersolubility of a group with seminormal 2-maximal subgroups was proved. In [10] first two authors obtained the sufficient conditions for the supersolubility of G under the condition that all Sylow subgroups or all maximal subgroups of two non-conjugate maximal subgroups of G are seminormal in G.

We introduce the new concept that unites subnormality and seminormality.

Definition. A subgroup A of a group G is called *semisubnormal* in G, if either A is subnormal in G, or is seminormal in G.

Let M and H be non-conjugate maximal subgroups of G. In the present paper we proved the supersolubility of a group G under the condition that all Sylow subgroups of M and H are semisubnormal in G. We also obtained the nilpotency of the second derived subgroup $(G')'$ of a group G under the condition that all maximal subgroups of M and H are semisubnormal in G.

1 Preliminary results

We use the standart terminology of [11], [12]. Recall that $A^G = \langle A^g \mid g \in G \rangle$ is the subgroup generated by all subgroups of G that are conjugate to A. Denote by $\pi(G)$ the set of all prime divisors of order of G and by $|G : A|$ the index of subgroup A in G. We use $N \lhd G$ to denote a normal subgroup N of G. For maximal subgroup M of G we will use the following notation: $M <\cdot G$. We write $O_p(G)$ to denote the greatest normal p-subgroups of G. The semidirect product of a normal subgroup A and a subgroup B is written as follows: $A \rtimes B$. A subgroup U is called subnormal in G, if there exist the subgroups $U_0, U_1, \ldots, U_s$ such that

$$U = U_0 \lhd U_1 \lhd \ldots \lhd U_{s-1} \lhd U_s = G.$$

Let $\mathfrak{F}$ be a non-empty formation. If G is a group then $G^{\mathfrak{F}}$ denotes the $\mathfrak{F}$-residual of G, that is the intersection of all those normal subgroups N of G for which $G/N \in \mathfrak{F}$. We define $\mathfrak{F} \circ \mathfrak{H} = \{G \mid G^{\mathfrak{H}} \in \mathfrak{F}\}$ and call $\mathfrak{F} \circ \mathfrak{H}$ the formation product of $\mathfrak{F}$ and $\mathfrak{H}$, see [13, IV, 1.7]. As usually, $\mathfrak{F}^2 = \mathfrak{F} \circ \mathfrak{F}$. A formation

$\mathfrak{F}$ is said to be saturated if $G/\Phi(G) \in \mathfrak{F}$ implies that $G \in \mathfrak{F}$. In this paper $\mathfrak{N}$, $\mathfrak{U}$ and $\mathfrak{A}$ denote the formations of all nilpotent, all supersoluble and all abelian groups respectively. The other definitions and terminology about formations could be referred to [11], [13], [14].

***Lemma* 1.1.** (1) *If H is a semisubnormal subgroup of G and $H \le X \le G$, then H is semisubnormal in X.*

(2) *If H is a semisubnormal subgroup of G and N is normal in G, then HN/N is semisubnormal in G/N.*

(3) *If H is a semisubnormal subgroup of G and Y is a non-empty set of elements from G, then*

$$H^Y = \langle H^y \mid y \in Y \rangle$$

is semisubnormal in G. In particular, H^g is semisubnormal in G for any $g \in G$.

Proof. If *H* is subnormal in *G*, then the statements (1)–(3) are true, see [11, Lemma 2.41, Theorem 2.43]. If *H* is seminormal, then this statements was proved in [8, Lemma 2]. Thus the statements (1)–(3) are true.

***Lemma* 1.2.** (1) *Let p be the greatest in $\pi(G)$ and P be a Sylow p-subgroup of G. If P is semisubnormal in G, then P is normal in G.*

(2) *If any Sylow subgroup of G is semisubnormal in G, then G is supersoluble.*

(3) *Let H be a maximal subgroup of G. If H is semisubnormal in G, then the index of H in G is a prime.*

(4) *If every maximal subgroup of G is semisubnormal in G, then G is supersoluble.*

(5) *If the index of H in G is a prime, then H is semisubnormal in G.*

Proof. (1) It is clear that if *P* is subnormal in *G*, then *P* is normal in *G*. If *P* is seminormal in *G* and *p* is greatest in $\pi(G)$, then by [7, Lemma 4], *P* is normal in *G*.

(2) Suppose that *G* has at least one subnormal Sylow subgroup *P*. Then *P* is normal in *G* and therefore is seminormal in *G*. Hence any Sylow subgroup of *G* is seminormal in *G*. By [7, Corollary 6], *G* is supersoluble.

(3) If *H* is subnormal in *G*, then *H* is normal in *G* and by [11, Lemma 3.17 (6)], $|G:H|$ is prime. Let *H* be a seminormal subgroup in *G* and *K* be a subgroup of *G* such that $HK = G$ and HK_1 is a proper subgroup of *G* for every proper subgroup K_1 of *K*. Let prime *r* divides the index $|G:H|$ and *R* be a Sylow *r*-subgroup of *K*. Then $HR = G$ and $G = H\langle x \rangle$ for $x \in R \setminus H$. We choose an element *x* such that its order is the smallest. Then $H\langle x^r \rangle = \langle x^r \rangle H = H$ and $|G:H| = r$.

(4) Let *M* be a maximal subgroup of *G*. By (3), the index of *M* in *G* is a prime. By [12, VI.9.2 (2)], *G* is supersoluble.

(5) Let $|G:H| = r$ and *R* be a Sylow *r*-subgroup of *G*. Then *R* is not contained in *H* and there exists an element $x \in R \setminus H$. Let $|x| = r^a$ and $|\langle x \rangle \cap H| = r^{a_1}$. It is obvious that $a > a_1$, hence

$$|\langle x \rangle H| = \frac{|\langle x \rangle||H|}{|\langle x \rangle \cap H|} = \frac{r^a \frac{|G|}{r}}{r^{a_1}} \ge |G|, \ \langle x \rangle H = G.$$

Now x^r belongs to *H* and *H* is seminormal in *G*, and therefore is semisubnormal in *G*.

***Lemma* 1.3.** (1) *If A is a semisubnormal 2-nilpotent subgroup of G, then A^G is soluble.*

(2) *Let p be the smallest prime divisor of order of G. If A is semisubnormal in G and p does not divide the order of A, then p does not divide the order of A^G.*

Proof. (1) If *A* is subnormal in *G*, then by [11, Theorem 5.31], A^G is soluble. If *A* is seminormal in *G*, then A^G is soluble by [8, Lemma 10].

(2) If *A* is a subnormal p'-subgroup of *G*, then by [11], A^G is a p'-subgroup. If *A* is a seminormal p'-subgroup of *G*, then A^G is a p'-subgroup by [8, Lemma 11].

***Lemma* 1.4** [15, Lemma 6]. *Let G be a soluble group. Assume that $G \notin \mathfrak{U}$, but $G/K \in \mathfrak{U}$ for every non-trivial normal subgroup K of G. Then*:

(1) *G contains a unique minimal normal subgroup N, $N = F(G) = O_p(G) = C_G(N)$ for some $p \in \pi(G)$;*

(2) $Z(G) = O_{p'}(G) = \Phi(G) = 1$;

(3) *G is primitive; $G = N \rtimes M$, where M is maximal in G with trivial core*;

(4) *N is an elementary abelian subgroup of order p^n, $n > 1$;*

(5) *if V is a subgroup G and $G = VN$, then $V = M^x$ for some $x \in G$.*

***Lemma* 1.5.** *Let $\mathfrak{F}$ be a formation. Then $\mathfrak{N} \circ \mathfrak{F}$ is a saturated formation.*

Proof. According to [14], the product $\mathfrak{N} \circ \mathfrak{F}$ is a local formation. Since saturated formation and local formation are equivalent concepts, $\mathfrak{N} \circ \mathfrak{F}$ is a saturated formation.

***Lemma* 1.6.** *Let $\mathfrak{F}$ be a saturated formation and G be a group. Assume that $G \notin \mathfrak{F}$, but $G/N \in \mathfrak{F}$ for all non-trivial normal subgroups N of G. Then G is a primitive group.*

Proof. Since $\mathfrak{F}$ is a saturated formation, it follows that $\Phi(G) = 1$ and *G* contains a unique minimal normal subgroup *N*. For some maximal subgroup *M* of *G*, we have $G = NM$, because $\Phi(G) = 1$. It is obvious that the core $M_G = 1$. Hence *G* is a primitive group.

***Lemma* 1.7** [11, Theorem 4.40–4.42]. *Let G be a soluble primitive group and M is a primitivator of G. Then the following statements hold*:

(1) $\Phi(G)=1$;

(2) $F(G)=C_G(F(G))=O_p(G)$ *and* $F(G)$ *is an elementary abelian subgroup of order* p^n *for some prime p and some positive integer n*;

(3) *G contains a unique minimal normal subgroup N and moreover*, $N=F(G)$;

(4) $G=F(G)\rtimes M$ *and* $O_p(M)=1$.

***Lemma* 1.8** [18]. *Let G be a minimal non-supersoluble group. Then the following holds*:

(1) *G is soluble*;

(2) *G contains a unique normal Sylow subgroup P and* $P=G^{\mathfrak{U}}$;

(3) $P/\Phi(P)$ *is a minimal normal subgroup of* $G/\Phi(P)$ *such that* $|P/\Phi(P)|>p$.

2 Supersolubility of a group with semisubnormal Sylow subgroups of two maximal subgroups

***Lemma* 2.1.** *Let M be a maximal subgroup of G. If all Sylow subgroups of M is semisubnormal in G, then* $|G:M|$ *is a prime, M and* G/M_G *is supersoluble. In particular, G is soluble.*

Proof. First we prove that *G* is soluble. We use induction on the order of *G*. Let *R* be an arbitrary Sylow subgroup of *M*. By Lemma 1.1, *R* is semisubnormal in *M*. Because it is true for any Sylow subgroup of *M*, it follows that *M* is supersoluble by Lemma 1.2 (2). In particular, *M* is 2-nilpotent. Hence every subgroup of *M* is also 2-nilpotent. By Lemma 1.3 (1), R^G is soluble. If $MR^G=G$, then *G* soluble, since $G/R^G=MR^G/R^G\cong M/M\cap R^G$ is supersoluble. Let $R^G\le M$. Hence G/R^G has a maximal subgroup M/R^G. Let S/R^G be a Sylow *t*-subgroup of M/R^G and *T* be a Sylow *t*-subgroup of *S*. By [11, Theorem 1.65], TR^G/R^G is a Sylow *t*-subgroup of S/R^G. Then $S=TR^G$ and *T* is a Sylow *t*-subgroup of *M*. By hypothesis, *T* is semisubnormal in *G* and by Lemma 1.1, $TR^G/R^G=S/R^G$ is semisubnormal in G/R^G. Then by induction, G/R^G is soluble, consequently *G* is soluble. So the solubility of *G* is proved.

We use induction on the order of *G* and prove that G/M_G is supersoluble. If $M_G\ne 1$, then M/M_G is a maximal subgroup of G/M_G. As in the previous indent it is easy to verify that the quotient G/M_G with maximal subgroup M/M_G satisfy all conditions of the lemma. By induction, $(G/M_G)/(M/M_G)_{G/M_G}$ is supersoluble. Since $(M/M_G)_{G/M_G}=1$, it follows that G/M_G is supersoluble and $|G/M_G:M/M_G|=|G:M|$ is a prime.

Therefore we consider that $M_G=1$. Now *G* is primitive and $G=N\rtimes M$, where *N* is a *r*-subgroup. Since *M* is supersoluble, it follows that $M=P\rtimes T$, where $P=M_p$ is a Sylow *p*-subgroup for the greatest $p\in\pi(M)$. Let $p=r$. Then $O_p(M)\ne 1$, a contradiction. Hence $p\ne r$ and *P* is a Sylow *p*-subgroup of *G*. Suppose that *P* is subnormal in *G*. Then $P\lhd G$, a contradiction. Consequently *P* is seminormal in *G*. Now *G* has a subgroup *U* such that $G=PU$. It is clear that $N\le U$. Let *x* be an elementof prime order that lies in *N*. Then $P\langle x\rangle\le G$. If $p>r$, then $P\lhd P\langle x\rangle$. Thus $P\lhd\langle M,x\rangle=G$, a contradiction. If $p<r$, then *N* is a Sylow *r*-subgroup of *G*, since *p* is the greatest in $\pi(M)$. Now all Sylow subgroups of *G* is semisubnormal in *G*. By Lemma 1.2 (2), *G* is supersoluble. Hence $|G:M|$ is a prime. The lemma is proved. □

***Remark* 2.1.** Soluble groups containing a supersoluble subgroup of prime index were studied in [16], [17].

***Theorem* 2.1.** *Suppose that G has two non-conjugate maximal subgroups H and K. If all Sylow subgroups of H and of K are semisubnormal in G, then G is supersoluble.*

Proof. We use induction on the order of *G*. By Lemma 2.1, *G* is soluble, *H* and *K* are supersoluble. Besides, quotients G/H_G and G/K_G are supersoluble. In particular, indices of subgroups *H* and *K* in *G* are primes. By Lemma 1.2 (5), subgroups *H* and *K* are semisubnormal in *G*.

Let *N* be an arbitrary non-trivial normal subgroup in *G*. If *N* is not contained in $H\cap K$, then *N* is either not contained in *H*, or *N* not contained in *K*. If *N* is not contained in *H*, then $HN=G$ and

$$G/N=HN/N\cong H/H\cap N$$

is supersoluble. Similarly, if *N* is not contained in *K*, then $KN=G$ and G/N is supersoluble. Let $N\le H\cap K$. Then $G/N=(H/N)(K/N)$. Let $\overline{R}$ be a Sylow *r*-subgroup of H/N. Then *H* has a Sylow *r*-subgroup *R* such that $\overline{R}=RN/N$. By hypothesis, *R* is semisubnormal in *G*. By Lemma 1.1 (2), $\overline{R}=RN/N$ is semisubnormal in G/N. Similarly, every Sylow subgroup of K/N is semisubnormal in G/N. By induction, G/N is supersoluble.

So, in any case G/N is supersoluble. By Lemma 1.6, *G* is primitive and statements (1)–(5) of the Lemma 1.4 are true. In particular, $|N|=p^n>p$. If $N\not\le H$, then $G=N\rtimes H$. Since *H* is semisubnormal in *G*, then by Lemma 1.2 (5), $|N|=|G:H|$ is a prime, a contradiction. Similarly, in the case when $N\not\le K$. Hence we consider that $N\le H\cap K$. Because *H* and *K* are supersoluble and $N=C_G(N)$, we have p is the greatest in $\pi(H)$ and in $\pi(K)$, hence p is the greatest in $\pi(G)$. Since $O_p(G/N)=1$ and G/N is supersoluble, p does not divide the order of G/N and N is a Sylow p-subgroup of G.

Let $N_1 \le N$, $|N_1| = p$ and R be a Sylow r-subgroup of M. Since $M = G_{p'} = H_{p'}K_{p'}$, it follows that $R = H_rK_r$ for some Sylow r-subgroups H_r and K_r of H and of K respectively. By hypothesis, subgroups H_r and K_r are semisubnormal in G. If H_r is subnormal in G, then by [14, Corollary 7.7.2 (1)], $H_r \le O_r(G) \le O_{p'}(G) = 1$. Similarly, if K_r is subnormal in G, then $K_r \le O_{p'}(G) = 1$. Consequently H_r and K_r are semisubnormal in G. Hence there exists a subgroup U such that $G = H_rU$ and H_r is permutable with any subgroup of U. Since $N \le U$, we have H_r is permutable with N_1. Similarly, K_r is permutable with N_1. Hence R is permutable with N_1. It is true for any $r \in \pi(M)$. Therefore M is permutable with N_1. Now MN_1 is a subgroup of G and N_1 is normal in MN_1. Since N is abelian, N_1 is normal in $NM = G$, a contradiction with $|N| > p$. The theorem is proved. □

***Example* 2.1.** The group $G = PSL(2,5)$ has maximal subgroups $H = Z_3 \rtimes Z_2$ and $K = Z_5 \rtimes Z_2$. Maximal subgroups of Sylow subgroups of H and K are trivial, hence are semisubnormal in G, but G is not soluble. Therefore the semisubnormality of maximal subgroups of Sylow subgroups of H and K under the conditions of Theorem 2.1 is not sufficient condition for the solubility of G.

***Corollary* 2.1.1** [10, Theorem E]. *Suppose that G has two non-conjugate maximal subgroups H and K. If all Sylow subgroups of H and of K are seminormal in G, then G is supersoluble.*

3 On a group with semisubnormal maximal subgroups of two maximal subgroups

***Lemma* 3.1.** *Let M be a maximal subgroup of G. If all maximal subgroups of M are semisubnormal in G, then G is soluble.*

Proof. We use induction on the order of G. Let K be a maximal subgroup of M. By hypothesis, K is semisubnormal in G and by Lemma 1.1 (1), K is semisubnormal in M. By Lemma 1.2 (4), M is supersoluble and consequently is 2-nilpotent. Then K is also 2-nilpotent and by Lemma 1.3, K^G is soluble. Since M is a maximal subgroup of G, then either $MK^G = G$, or $K^G \le M$. If $MK^G = G$, then G is soluble. Let $K^G \le M$. Then M/K^G is a maximal subgroup of G/K^G. Let $\overline{S}$ be a maximal subgroup of M/K^G. Then M has a maximal subgroup S such that $K^G \le S$ and $\overline{S} = S/K^G$. By hypothesis, S is semisubnormal in G. By Lemma 1.1, SK^G/K^G is semisubnormal in G/K^G. Since $K^G \le S$, we have $S = SK^G$ and S/K^G is semisubnormal in G/K^G. By induction, G/K^G is soluble. Then G is soluble. The lemma is proved. □

***Example* 3.1.** In the condition of the Lemma 3.1, the index $|G:M|$ may not be a prime. For example, the group $G = A_4 = A \rtimes B$. The subgroup B has the order 3. Besides, B is maximal in G and all maximal subgroups of B are semisubnormal in G, but $|G:B| = 4$ is not a prime.

***Example* 3.2.** The alternating group $G = A_4$ of degree 4 has two non-conjugate maximal subgroups $A = Z_3$ and $B = Z_2 \times Z_2$. It is clear that all maximal subgroups of A and of B are semisubnormal in G. But G is non-supersoluble.

***Theorem* 3.1.** *Let H and K are non-conjugate maximal subgroups of G. If all maximal subgroups of H and of K are semisubnormal in G, then the second derived subgroup* $(G')'$ *is nilpotent.*

Proof. Note that the nilpotency of the second derived subgroup $(G')'$ is equivalent to $G \in \mathfrak{N} \circ \mathfrak{A}^2$.

Assume that the claim is false and let G be a minimal counterexample. By Lemma 3.1, G is soluble. By Lemma 1.1 (1), every maximal subgroup of H is semisubnormal in H and by Lemma 1.2 (4), H is supersoluble. Similarly, K is supersoluble.

Let N be an arbitrary non-trivial normal subgroup in G. Then either $HN = G$, or $HN = H$. If $HN = G$, then

$$G/N = HN/N \cong H/H \cap N \in \mathfrak{N} \circ \mathfrak{A} \subseteq \mathfrak{N} \circ \mathfrak{A}^2.$$

If $HN = H$, then $N \le H$. Similarly either $KN = G$ and $G/N \in \mathfrak{N} \circ \mathfrak{A}^2$, or $N \le K$. Let $N \le H \cap K$. Then G/N has non-conjugate maximal subgroups H/N and K/N. If $\overline{S}$ is a maximal subgroup of H/N, then H has a maximal subgroup S such that $\overline{S} = S/N$. By hypothesis, S is semisubnormal in G and by Lemma 1.1 (2), $\overline{S} = S/N$ is semisubnormal in G/N. Similarly, if $\overline{T}$ is a maximal subgroup of K/N, then it is semisubnormal in G/N. Therefore for G/N with non-conjugate maximal subgroups H/N and K/N the conditions of the theorem are satisfied. By induction, $G/N \in \mathfrak{N} \circ \mathfrak{A}^2$.

By Lemmas 1.5 and 1.6, G is primitive. Then for G we have Lemma 1.7. Hence $\Phi(G) = 1$ и G contains a unique minimal normal subgroup N such that $N = C_G(N)$.

Suppose that at least one of the subgroups H or K is normal in G. For example, let H be normal in G. Then $|G:H| = q$ and by [16, Theorem 1], $G = N \rtimes T$, where T has abelian subgroup of index q. Since $T \in \mathfrak{A}^2$, it follows that $G \in \mathfrak{N} \circ \mathfrak{A}^2$, a contradiction.

Therefore in the future we assume that the subgroups H and K are non-normal. By [16, Theorem 2], $G = N \rtimes T$, where

$$T / C_T(N) \cong \overline{T} = < y > \rtimes (< t >< z >)$$

and $< z >= Z(\overline{T})$. Since $N = C_G(N)$, we have $C_T(N) = 1$ and $T = (< y > \times < z >) < t >$, because $< z >= Z(T)$. Thus $T \in \mathfrak{A}^2$ and $G \in \mathfrak{N} \circ \mathfrak{A}^2$, a contradiction. The theorem is proved. □

***Corollary* 3.1.1.** *If all 2-maximal subgroups of G are semisubnormal in G, then the derived subgroup G' is nilpotent.*

Proof. Note that the nilpotency of the derived subgroup G' is equivalent to $G \in \mathfrak{N} \circ \mathfrak{A}$.

Assume that the claim is false and let G be a minimal counterexample. It is easy to show that G / N satisfies the hypothesis of the corollary, where N is an arbitrary non-trivial normal subgroup of G. By induction, $G / N \in \mathfrak{N} \circ \mathfrak{A}$. Hence by Lemmas 1.5 and 1.6, G is primitive.

Let M be an arbitrary maximal subgroup of G. Then by Lemmas 1.1 (1) and 1.2 (4), M is supersoluble. Hence either G is supersoluble, or G is a minimal non-supersoluble group.

If G is supersoluble, then $G \in \mathfrak{N} \circ \mathfrak{A}$ by [11, Theorem 4.52], a contradiction.

Let G be a minimal non-supersoluble group. By Lemmas 1.7 and 1.8, G is soluble, P is a unique minimal normal subgroup of G, $|P| > p$ and P is a Sylow p-subgroup of G such that $G = P \rtimes M$, where M is a maximal subgroup of G. Besides, M is a Hall p'-subgroup of G. Let P_1 be a subgroup of prime order p of P.

If M is abelian, then $G \in \mathfrak{N} \circ \mathfrak{A}$, a contradiction. Therefore we assume that M is non-abelian. Hence M has maximal subgroups M_1 and M_2 such that $M = \langle M_1, M_2 \rangle$. If at least one of the subgroups M_1 or M_2 is subnormal in G, then $O_{p'}(G) \neq 1$, a contradiction. Thus M_1 and M_2 are seminormal in G. Hence there are the subgroups V_1 and V_2 such that

$$M_1V_1 = M_2V_2 = G, M_1P_1 = P_1M_1, M_2P_1 = P_1M_2,$$

because $P \leq V_1 \cap V_2$. Then $M_1 \leq N_G(P_1)$ and $M_2 \leq N_G(P_1)$. Therefore $P_1 \lhd G = P\langle M_1, M_2 \rangle$, a contradiction. The corollary is proved. □

REFERENCES

1. *Su*, *X.* On semi-normal subgroups of finite group / X. Su // J. Math. (Wuhan). – 1988. – Vol. 8, № 1. – P. 7–9.

2. *Wang*, *P. C.* Some sufficient conditions of a nilpotent group / P. C. Wang // Journal of Algebra. – 1992. – Vol. 148, № 2. – P. 289–295.

3. *Foguel*, *T.* On seminormal subgroups / T. Foguel // Journal of Algebra. – 1994. – Vol. 165. – P. 633–635.

4. *Carocca*, *A.* Some solvability criteria for finite groups / A. Carocca, H. Matos // Hokkaido Math. J. – 1997. – Vol. 26. – P. 157–161.

5. *Podgornaya*, *V.V.* Seminormal subgroups and supersolubility of finite groups / V.V. Podgornaya // Vesti Akad. Navuk Belarusi Ser. Fiz.-Mat. Navuk. – 2000. – Vol. 4, № 4. – P. 22–25 (Russian).

6. *Podgornaya*, *V.V.* Supersolubility of a finite group with seminormal second maximal subgroups / V.V. Podgornaya // Francisk Scorina Gomel State University proceedings. – 2008. – Vol. 47, № 2. – P. 154–156 (Russian).

7. *Monakhov*, *V.S.* Finite groups with a seminormal Hall subgroup / V.S. Monakhov // Math. Notes. – 2006. – Vol. 80, № 4. – P. 542–549.

8. *Knyagina*, *V.N.* Finite groups with seminormal Schmidt subgroups / V.N. Knyagina, V.S. Monakhov // Algebra and Logic. – 2007. – Vol. 46, № 4. – P. 244–249.

9. *Guo*, *W.* Finite groups with seminormal Sylow subgroups / W. Guo // Acta Mathematica Sinica. – 2008. – Vol. 24, 10. – P. 1751–1758.

10. *Monakhov*, *V.S.* Finite groups with two supersoluble subgroups / V.S. Monakhov, A.A. Trofimuk // J. Group Theory. – 2019. – Vol. 22. – P. 297–312.

11. *Monakhov*, *V. S.* Introduction to the Theory of Finite groups and their Classes / V.S. Monakhov. – Minsk: Vyshejshaja shkola, 2006. – 207 p. (Russian).

12. *Huppert*, *B.* Endliche Gruppen I / B. Huppert. – Berlin – Heidelberg – New York: Springer, 1967. – 796 p.

13. *Doerk*, *K.* Finite soluble groups / K. Doerk, T. Hawkes. – Berlin – New York: Walter de Gruyter, 1992. – 892 p.

14. *Shemetkov*, *L.A.* Formations of finite groups / L.A. Shemetkov. – Moskow: Nauka, 1978. – 272 p. (Russian).

15. *Monakhov*, *V.S.* On the supersoluble residual of a product of subnormal supersoluble subgroups / V.S. Monakhov, I.K. Chirik // Siberian Math. J. – 2017. – Vol. 58, № 2. – P. 271–280.

16. *Kazarin*, *L.S.* Finite solvable groups with supersolvable maximal subgroups / L.S. Kazarin, Yu.A. Korzyukov // Soviet Math. (Iz. VUZ). – 1980. – Vol. 24, № 5. – P. 23–29 (Russian).

17. *Buzlanov*, *A.V.* Finite soluble groups with given maximal subgroups / A.V. Buzlanov // Voprosy algebry. – 1993. – Vol. 6. – P. 35–45 (Russian).

18. *Doerk*, *K.* Minimal nicht überauflösbare, endliche Gruppen / K. Doerk // Math. Z. – 1966. – Vol. 91. – P. 198–205.

Поступила в редакцию 29.05.19.