To the Problem of Compton Rotation of the Plane of Polarization of X-photons in Magnetic Field

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Abstract

In relativistic approach using electronic propagator the difference of Compton forward scattering amplitudes in quantizing magnetic field is calculated for electron in the ground state and circularly polarized hard X-photons moving at arbitrary angle to magnetic field with opposite helicities. A formula is obtained for the calculation of Compton rotation angle of the plane of linear polarization of photons per unit path in electron gas with high degree of spin polarization of electrons.

1 Introduction

In present work the research is done according to the suggestion of V.G. Baryshevsky and V.V. Tikhomirov.

The effect of Compton rotation of the plane of polarization of hard X- (soft gamma-) photons in the absence of magnetic field was theoretically predicted by V.G. Baryshevsky and V.L. Luboshitz in 1965 and experimentally tested on iron at early 1970s; the degree of electron spin polarization didn't exceed 8% with the peak of rotation observed in hard X-region [1, 2, 3, 4]. At the presence of quantizing magnetic field (so far unachievable in terrestrial conditions) the degree of electron spin polarization is expected to be close to 100%. But there comes a possibility of resonant absorption of a photon at electron transitions between Landau levels, though at $\hbar\omega >> 2\mu_B B$ or $\hbar\omega << 2\mu_B B$ (μ_B is Bohr magneton)

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such an absorption can be neglected. Hard X- and soft gamma-region (0,1–5 MeV) satisfy the 2d case at magnetic fields near the poles of magnetars ($\sim 10^{14} - 10^{15}$ Gs). In such a situation almost all the electrons are on the ground Landau level. The effect can influence the radiation spectra of superdense astrophysical objects with ultrastrong magnetic fields.

Let's consider a photon moving in an electron gas at arbitrary angle to magnetic field strength lines at the conditions pointed above. Let's compare Compton scattering forward amplitudes for circular polarization. These amplitudes will be used in the calculation of the angle of Compton rotation of the plane of polarization of photon. The approach applied here was used for the calculation of resonant Compton cross section in magnetic field [5]. In papers abroad [6, 7] either nonrelativistic approach was used, or the general expression of matrix element was not calculated up to the end, or the case of resonant scattering was considered also. Like in [5], it was mentioned that total accurate integration cannot be done in the general case.

2 Characteristics of photon, electron and external magnetic field

For constant homogeneous magnetic field directed along z axis let's choose the gauge of vector potential:

$$A_0 = A_x = A_z = 0, A_y = Bx. \tag{1}$$

If p_z is electron momentum along z axis, m is its mass, then one can write the expression for energy regarding Landau quantization; the electron wave functions are expressed in terms of bispinors u_n and Hermitian functions

H_n (i is imaginary unit)[5]:

$$\Psi_n(x) = A_n[i\sqrt{2eBc\hbar}U_n(x) + c^2(m+\sigma\tilde{m}_n)U_{n-1}(x)\gamma_1]u_n, n \geq 0,$$
$$U_n(x) = \frac{1}{\sqrt{2}}\exp(-\frac{x^2}{2})H_n(x), H_n(x) = (-1)^n\exp(x^2)\frac{d^n}{dx^n}\exp(-x^2),$$
$$A_0 = \frac{1}{\sqrt{2c\hbar\varepsilon_0(\varepsilon_0+mc^2)\sqrt{Bec\hbar}}} \ (\sigma=-1),$$
$$A_n^2 = \sqrt{\frac{eB}{c\hbar}}\frac{\varepsilon_n+\sigma\tilde{m}_nc^2}{4c^4p_z^2\tilde{m}_n\varepsilon_n(\tilde{m}_nc^2+mc^2\sigma)}, \varepsilon_n = \sqrt{\tilde{m}_n^2c^4+p_z^2c^2},$$
$$\tilde{m}_nc^2 = \sqrt{m^2c^4+2nBec\hbar}, u_n = \begin{bmatrix} 0 & \sigma\tilde{m}_nc^2-\varepsilon_n & 0 & p_zc\end{bmatrix}^T. \quad (2)$$

Transposing is denoted by T; Dirac matrices γ_k ($k = 0, 1, 2, 3$) are in standard presentation. For the construction of electron propagator the following matrices are used [5]:

$$\beta_1 = \frac{1}{2}(1+i\gamma_2\gamma_1), \beta_2 = \frac{1}{2}(1-i\gamma_2\gamma_1). \quad (3)$$

Without loss of generality, one can choose a coordinate system, in which the photon wavevector $\vec{k}$ is in one of the coordinate planes. The angle between $\vec{k}$ and $\vec{B}$ is denoted by θ. Subject to conservation laws, at elastic forward scattering $\vec{k}$ (and photon frequency ω), as well as electron momentum $\vec{p}$, do not change. Then the 4-momentum of initial and final photon can be written in 2 ways:

$$k_0 = \frac{\omega}{c}, k_x = 0, k_y = \frac{\omega}{c}\sin\theta, k_z = \frac{\omega}{c}\cos\theta, \quad (4)$$
$$k_0 = \frac{\omega}{c}, k_x = \frac{\omega}{c}\sin\theta, k_y = 0, k_z = \frac{\omega}{c}\cos\theta. \quad (5)$$

Like in the absence of magnetic field, the momenta of virtual intermediate electron for R- and S-processes are expressed, respectively (subject to (4),(5)), by the formulas [5]:

$$\vec{g} = \vec{p} + \hbar\vec{k} \Rightarrow cg_0 = \varepsilon_0 + \hbar\omega, cg_2 = p_yc + \hbar k_y, cg_3 = p_zc + \hbar\omega\cos\theta,$$
$$\vec{f} = \vec{p} - \hbar\vec{k} \Rightarrow cf_0 = \varepsilon_0 - \hbar\omega, cf_2 = p_yc - \hbar k_y, cf_3 = p_zc - \hbar\omega\cos\theta. \quad (6)$$

According to [5], one also needs the denotation for coordinates (p_y is y-component of the momentum of initial and final electron; $k = 1, 2$):

$$j_k(p) = \sqrt{\frac{eB}{c\hbar}}(x_k + \frac{cp}{eB}), \xi_k = j_k(p_y), \rho_k = j_k(g_2), \eta_k = j_k(f_2). \quad (7)$$

Electron propagator contains structures of the following type [5]:

$$Y = \gamma_0\lambda_0 - \gamma_3\lambda_3 + mc, G_B(\lambda, x) = \sqrt{\frac{Be}{c\hbar}} \sum_{n=0}^{\infty} \frac{\hbar c^2}{c^2\lambda_0^2 - \varepsilon_n^2 - i \cdot 0} D,$$

$$D = U_n(x_1)U_n(x_2)Y\beta_1 + (1-\delta_{0n})U_{n-1}(x_1)U_{n-1}(x_2)Y\beta_2 +$$

$$+(1-\delta_{0n})i\sqrt{\frac{2neB\hbar}{c}}(U_{n-1}(x_1)U_n(x_2)\gamma_1\beta_1 - U_n(x_1)U_{n-1}(x_2)\beta_1\gamma_1). \quad (8)$$

3 Rearrangements of matrix elements

In [5] the matrix element of the process is presented, though the integration over coordinates is not done ad finem:

$$\Omega = \frac{-i\alpha(2\pi\hbar)^4 ce_\mu e_\nu'^*}{\hbar V S_0 \sqrt{\omega\omega'}} \delta^3(\vec{p} + \hbar\vec{k} - \vec{p'} - \hbar\vec{k'})\chi_{\mu\nu}, \omega = \omega',$$

$$\chi_{\mu\nu} = L(Q_{\mu\nu}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dx_1 dx_2 \bar{\Psi}_0(\xi_1) Q_{\mu\nu} \Psi_0(\xi_2). \quad (9)$$

Here $\alpha = e^2/\hbar c; \omega, \omega', e_\mu, e_\nu'$ are the frequencies and polarization vector components of initial and final photon, respectively; V is normalization volume for photon, S_0 is normalization area for electron in (xy) plane. Besides, subject to conservation laws at forward scattering some simplifications have been made (because in case of a difference between the components of the initial and final electron momentum along y axis a generalized coordinate containing p_y' instead of p_y was used instead of ξ_2 in [5]); δ-function includes y-, z- and t-components. If $\Delta x = x_2 - x_1$, then the general expression for $Q_{\mu\nu}$, according to [5], at elastic scattering to zero angle has the form (subject to (6)–(8)):

$$Q_{\mu\nu} = \exp(ik_x\Delta x)\gamma_\nu G_B(g, \rho)\gamma_\mu + \exp(-ik_x\Delta x)\gamma_\mu G_B(f, \eta)\gamma_\nu. \quad (10)$$

From (6), (7) one can see, that the first term in (10) corresponds to R-process, the second – to S-process. Plugging (10) at $\mu, \nu = \overline{1,2}$ into (9), one can show that it gives zero at the convolution of u_0 (2) with the combinations of Dirac matrices in (8), which don't contain β_2 (3). At other combinations of μ, ν there are nonzero convolutions with matrix combinations $\beta_1\gamma_1$ and $\gamma_1\beta_1$, but they are mutually annihilated either because of the opposite signs in (8), or due to subtractions in (13). For the summands with β_2

$$\gamma_1\zeta\gamma_2 = -\gamma_2\zeta\gamma_1, \zeta = \gamma_0\beta_2, \gamma_3\beta_2, \beta_2. \qquad (11)$$

If one chooses the formulas (4), imaginary exponents disappear in (10), and in the choice of (5), according to (6),(7), $\rho_k = \eta_k = \xi_k, k = \overline{1,2}$. So, one can integrate in different ways in (9), with different simplifications in each case. Further reflection is for the first case, i.e. when photon is moving in (yz) plane (in the second case the final result is the same).

Writing out the components of polarization vector for circular case involves some arbitrariness (compare [1, 8, 9]). Choosing right (+) or left (-) circular polarization, according to [9], one can write (in 3-dimensional form zero component is omitted):

$$e_{(\pm)} = \frac{1}{\sqrt{2}}\left[\begin{array}{cccc} 0 & \mp i & \cos\theta & -\sin\theta \end{array}\right]^T. \qquad (12)$$

Substituting (12) into (9), subject to (11) one finds the difference of matrix elements:

$$\frac{\Omega_{(+)} - \Omega_{(-)}}{2c\alpha(2\pi\hbar)^4} = \frac{\delta^3(\vec{p} + \hbar\vec{k} - \vec{p'} - \hbar\vec{k'})\cos\theta}{\hbar V S_0 \omega}\tilde{\chi}, \tilde{\chi} = L(\tilde{Q}_{21}). \qquad (13)$$

The difference of $\tilde{Q}_{21}$ from Q_{21} is that only summands containing β_2 remain in (8) making substitution into (10). If $k_x = 0$ in (10), then one writes out the integral in (13), concerning (8) and (11):

$$\tilde{\chi} = cA_0^2(cBe\hbar)^{3/2}\sum_{n=1}^{\infty}(J_n(g)I_{n-1}^2(\rho)\Lambda(g) - J_n(f)I_{n-1}^2(\eta)\Lambda(f)),$$
$$J_n(\lambda) = (c^2\lambda_0^2 - \varepsilon_n^2 - i\cdot 0)^{-1}, \Lambda(\lambda) = \Gamma_0\lambda_0 - \Gamma_3\lambda_3 + \Gamma mc,$$
$$\Gamma_0 = \bar{u}_0\gamma_1\gamma_0\beta_2\gamma_2 u_0 = -2i\varepsilon_0 K, \Gamma_3 = \bar{u}_0\gamma_1\gamma_3\beta_2\gamma_2 u_0 = -2ip_z cK,$$
$$\Gamma = \bar{u}_0\gamma_1\beta_2\gamma_2 u_0 = 2imc^2 K, K = mc^2 + \varepsilon_0,$$
$$I_{n-1}^2(\rho) = I_{n-1}^2(\eta) = \frac{\pi\hbar c}{2Be}\phi^{2n-2}\exp(-\frac{\phi^2}{2}), \phi = \frac{\hbar\omega\sin\theta}{\sqrt{cBe\hbar}}. \qquad (14)$$

4 The calculation of rotation angle of the plane of linear polarization per unit path

Let's write down the formulae for Compton rotation angle of the plane of linear polarization of hard X-photon per unit path [1]:

$$\frac{d\varphi}{dl} = \frac{2\pi n_e c}{\omega}(\vec{q}\cdot\vec{n})ReW_2(\omega), \tag{15}$$

where n_e is electron density, $\vec{q}$ is their vector of spin polarization, $\vec{n} = \vec{k}/\left|\vec{k}\right|$. Our task is reduced to finding $W_2(\omega)$.

Denoting Compton forward scattering amplitude as F, let's write out the relationships for the case of of right (+) and left (–) circular photon polarization [1]:

$$F_{(\pm)} = W_1(\omega) \mp W_2(\omega)(\vec{q}\cdot\vec{n}), 2ReW_2(\omega)(\vec{q}\cdot\vec{n}) = Re(F_{(-)} - F_{(+)}). \tag{16}$$

Now one has to find the relationship between $F_{(\pm)}$ in (16) and $\Omega_{(\pm)}$ in (9), (13). The same quantities are denoted differently in different references, which can be demonstrated by comparison of the formulae in [1] and [8], for example, where the same amplitude is present, the role of which can be played by F in our case. Considering this circumstance and using the formulae in [8], let's write the analog of the expression (9) in the absence of magnetic field:

$$\Omega = 2\pi\varepsilon\frac{i(2\pi\hbar)^4 c\hbar F}{V^2\sqrt{\varepsilon_0\hbar\omega\varepsilon_0'\hbar\omega'}}\delta^3(\vec{p}+\hbar\vec{k}-\vec{p'}-\hbar\vec{k'})\delta(p_x+\hbar k_x-p_x'-\hbar k_x'). \tag{17}$$

Herewith ε is total energy in the center-of-mass system; ε_0' is the energy of final electron. In our case, according to (2), subject to the conservation laws

$$\varepsilon = \varepsilon_0 + \hbar\omega, \varepsilon_0 = \sqrt{m^2c^4 + p_z^2c^2} = \varepsilon_0', \omega' = \omega. \tag{18}$$

As F can be expressed from (17) through Ω, the expression for F changes in the presence of magnetic field, because the right part of (9) is substituted into the left part of (17). Let's make a substitution for the x-component of δ-function in (17) by analogy to [8]:

$$\delta(p_x+\hbar k_x-p_x'-\hbar k_x') \to \frac{L_0}{2\pi\hbar} = \frac{V}{2\pi\hbar S_0}. \tag{19}$$

Then subject to (13) and (16) one obtains:

$$W_2(\omega)(\vec{q}\cdot\vec{n}) = \frac{\pi\hbar c^2\alpha\cos\theta}{2(\varepsilon_0+\hbar\omega)}\exp(-\frac{\hbar\omega^2\sin^2\theta}{2cBe})\Phi,$$

$$\Phi = \sum_{n=1}^{\infty}(\frac{\hbar\omega^2\sin^2\theta}{cBe})^{n-1}(\Xi_n(g)-\Xi_n(f)), \Xi_n(\lambda) = \frac{\lambda_0\varepsilon_0-\lambda_3 p_z c-m^2c^3}{c^2\lambda_0^2-\varepsilon_n^2-i\cdot 0}. \quad (20)$$

If photon's motion is along magnetic field strength lines, then $\theta = 0$, the cosine and the exponent in (20) are substituted by unity, and one term ($n = 1$) remains in the series. To obtain numerical results for the general case, one has to calculate the series and to average over momentum p_z. Besides, there is still an open question of the competition of the effect with photon splitting and merging in ultrastrong magnetic fields [8, 10].

5 Summary. The main results

In relativistic approach in the framework of quantum electrodynamics the difference of Compton forward scattering amplitudes in ultrastrong magnetic field is calculated for electron in the ground state and circularly polarized hard X-photons moving at arbitrary angle to magnetic field strength lines with opposite helicities. A corresponding formula is obtained for the calculation of Compton rotation angle of the plane of polarization of photons per unit path in electron gas with high degree of spin polarization of electrons.

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